

# Volume Optimization of Two-Stage Helical Gear Train using Differential Evolution Algorithm

Vikash Kumar<sup>1\*</sup> & Sachin Kumar Singh<sup>2</sup>

<sup>1</sup>Department of Mechanical Engineering, Indian Institute of Technology Patna, Bihta 801 106, Bihar, India

<sup>2</sup>Department of Mechanical Engineering, Indian Institute of Technology (ISM) Dhanbad, Dhanbad 826 004, Jharkhand, India

*Received 21 August 2023; revised 26 December 2023; accepted 15 January 2024*

In high-performance power transmission systems like automotive and aerospace, the proper gear train design is essential because it requires minimum weight and high-efficiency gearboxes with maximum service life. An iterative design method that takes into account all viable design options is used to achieve the desired outcome. This procedure cannot be automated using the traditional methods utilized in its design. As a result, this paper makes an attempt to automate the gear train's preliminary design. This paper uses the Differential Evolution (DE) optimization technique and a dynamic penalty function to optimize the two-stage helical gear train's design parameters by minimising the objective function i.e., the gear train's overall geometrical volume (size). The objective function is constrained by bending force, surface fatigue strength, and interference equations of helical gear train with the design variables such as number of teeth, face width, module, and helix angle of each gear. Ranges of design parameters are taken from the manufacturer's catalogue. The optimised design parameters obtained from the proposed approach are compared and validated with the standard gear parameters (i.e., catalogue value) and with the results published in the literature applying other optimising approaches such as Genetic Algorithm (GA) and Fminsearch Solver (FS). The proposed approach shows a significant reduction i.e., 18.51% with GA and 18.14% with FS in the overall geometrical volume (size) of the two-stage helical gear train as compared to the published work. The presented approach enhances the design optimization problem of gear train which may be used in automobile, aircrafts, and robotics application for optimal performance.

**Keywords:** Design optimization, Design parameters, Dynamic penalty function, Gear parameters, Power transmission

## Introduction

The process of designing a new product involves a number of variables and stages, which vary depending on the input data, level of design, design methodology, strategies, and outcomes. In the process of optimizing a mechanical design, certain goals like deflection, strength, wear, corrosion, and weight are constantly taken into account.<sup>1-4</sup> However, the objective function that results from design optimization for a whole mechanical assembly has a problematically high number of design variables. Therefore, it makes sense to use optimization techniques on specific assemblies instead of entire assembly. For instance, optimizing the gearbox is computationally and theoretically easier than optimizing the entire operation in an auto-mobile power transmission system. Gear design is still a current endeavor. Gears are used in numerous mechanical power transmission techniques, such as aerospace, auto-mobile, and computer instrumentation.

There are an enormous number of design parameters due to the complicated shape and geometry of gears.<sup>5-7</sup>

A typical gear design involves calculations based on the AGMA standard for interference, efficiency, teeth surface fatigue, teeth surface durability, and other variables in order to ensure that these gear trains can carry out the tasks specified in the design specification. Gear design requires empirical formulation, unique graphs, and tables, which results in a challenging design. On the basis of the aforementioned information, computer-aided gear design is needed because manual design is quite complex. Computer assistance allows for the iterative application of design as well as the selection of the design variables that best meet the requirements. The design thus acquired will not be the best one, as the design variables thus generated within the aforementioned method only satisfy one situation at a time, for instance, surface durability is calculated using the same module as is used to compute bending force. If it does not exceed the limits of surface durability, it is permitted; otherwise, it is altered as necessary. Therefore, it is

\* Author for Correspondence  
E-mail: 1821me18@iitp.ac.in

necessary to use optimization tools to look at design variables while also satisfying the requirements. Additionally, the need for small, effective, and safe gears is expanding, which compels machine designers to employ the optimal design practices. The trial-and-cut method is used to solve many problems in today's world, but it takes long time to find the best solution. A common machine component used in many sectors is the gear. It delivers energy with extraordinary precision. We typically employ the trial-and-cut method when developing an apparatus to evaluate a number of comparable parameters, including input power, rotation frequency, torsional strength, and bending strength. These methods, however, do not take into account the technique of maximizing center-to-center distance and gear weight. However, there are a lot of competing goals involved in solving engineering problems. In the past, designing a gear drive required an enormous number of calculations based on gear specification tactics, try and error techniques, etc. This process might be highly time-consuming.<sup>8</sup>

There have been numerous studies looking into how to use computers to optimize the gear. Senthilkumar *et al.*<sup>9</sup> have used Genetic Algorithm (GA) and Fminsearch Solver (FS) from Matlab toolbox for optimizing the two-stage helical gear drive. Using design variables like module, number of teeth on gears, face width, and helix angle, respectively, and by satisfying various constraints, first the volume optimization of a two-stage helical gear drive is done. Then, the load-carrying capacity of shafts is calculated using these design variables. Sanghvi *et al.*<sup>10</sup> have studied the same thing using three different optimization techniques i.e., Matlab optimization toolbox, GA, and NSGA-II. Number of gear teeth, module, and Face width were the mostly impacted parameters from per volume standpoint. Song *et al.*<sup>11</sup> also optimized the design parameters of two-stage helical gear trains using GA and Nastran optimization techniques. This paper minimized the volume using penalty function method under various constraint conditions. Huang *et al.*<sup>12</sup> have done the same thing for three-stage spur gear with adding one more objective i.e., maximum surface fatigue life in design objectives. By using the Simulated Annealing (SA) techniques and Particle Swarm Optimization (PSO), Savsani *et al.*<sup>8</sup> evaluated the least weight of a spur gear drive. In order to solve a Nonlinear Integer Programming (NIP) issue for constraints like shafts torsional strength, gear bending strength, and gear dimension, Yokota *et al.*<sup>13</sup> developed an improved

GA. The changes in gear weight and space area were used to prove the viability of the suggested approach. The volume of single and multistage spur gear units was reduced by Thompson *et al.*<sup>14</sup> using the same loading circumstances and other design criteria. Golab *et al.*<sup>15</sup> also did the optimization of gear drive based on minimum weight/volume design. Along with volume optimization, Mendi *et al.*<sup>16</sup> optimized the rolling bearing parameter for spur gear through GA. Further, Padmanabhan *et al.*<sup>17</sup> did the design optimization of the worn gear drive. Tong & Walton<sup>18</sup> optimized the volume and center distance of the internal gears by two methods i.e., half-section algorithm and belt zone search. By illustrating the trade-off between surface fatigue life and lowest volume, the method helps to extend conventional design processes.

In all the above literature surveys, various optimization techniques such as GA, NSGA-II, PSO, SA and other evolutionary algorithms are applied for optimizing the design parameters of the gear. Therefore, it is unclear from the literature review which optimization strategy is optimal for optimizing the design variables of the gear. In this work, the Differential Evolution (DE) optimizing technique along with dynamic penalty function<sup>11</sup> is used to optimize the design variable of two-stage helical gear train by minimizing the overall volume of the gear train. The mutation and recombination processes used by the DE algorithm are distinct from those used by other evolutionary algorithms. In a mutation operation, the target solution and the altered solution are combined to produce the trail vector by combining the differences between the two populations that were chosen at random. The best outcome is attained in accordance with the survival of the fittest theory. A comparative analysis is presented to demonstrate the validity of the proposed optimization technique with the previously developed technique.<sup>9</sup> The compared results show how the proposed technique provides the best optimal design variables with least overall volume of the two-stage helical gear train.

## Methodology

In this section, the optimization technique of two-stage helical gear train is developed, with design objectives of minimum overall volume and highest load-carrying capacity. Schematic representation of a two-stage helical gear train is presented in Fig. 1. The gear ratios between the gear pairs are selected so that their values are viable and the product remains

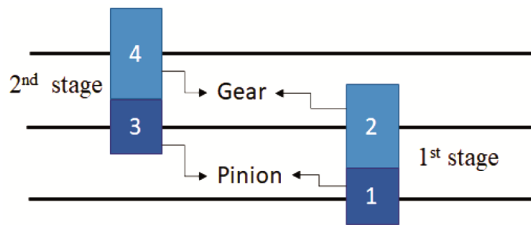


Fig. 1 — Two-stage helical gear train schematic

identical to that required. The gear and pinion are made from the same material in this design. The entire design process is based on the pinion, so the gear’s teeth are dependent on the pinion’s teeth and gear ratio. The methodology used for optimization is presented in Fig. 2. This flow chart gives the detailed explanations of the problem. The process steps for optimizing the two-stage helical gear train are also presented in flow chart. All the design parameters range are taken from the reference.<sup>9</sup> At the end, the DE optimized result is compared with the result presented in reference.<sup>9</sup>

**Design Parameters**

The gear's design variables that are often impacted from a volumetric perspective are the number of teeth, face width, and module. These variables affect the overall objectives either indirectly or directly. The design vector Y is

$$Y = [z_1, z_2, z_3, z_4, m_1, m_2, b_1, b_2, \psi] \dots (1)$$

where,  $z_1$  and  $z_3$  are the pinion’s teeth number,  $z_2$  and  $z_4$  are the gear’s teeth number,  $m_1$  and  $m_2$  are the module of 1<sup>st</sup> and 2<sup>nd</sup> stage of gear,  $b_1$  and  $b_2$  are the face width of 1<sup>st</sup> and 2<sup>nd</sup> stage of gear and  $\psi$  is helix angle of pinion and gear.

**Input Parameters**

The input parameters are key properties which are entered by the design engineer for accurate optimization. These are used for calculation of the objective function value and a lot of constraints. These input parameters are taken from <sup>9,19</sup> and shown in Table 1.

**Objective Function**

To begin the optimization process, first the volume of the double-stage helical gear train is minimized under the set constraints.<sup>9</sup> After that, the load carrying capacity of each stage is maximized once the optimal values for the design variables for the minimal volume have been reached. The lowest load carrying potential of these two stages is picked as the gear

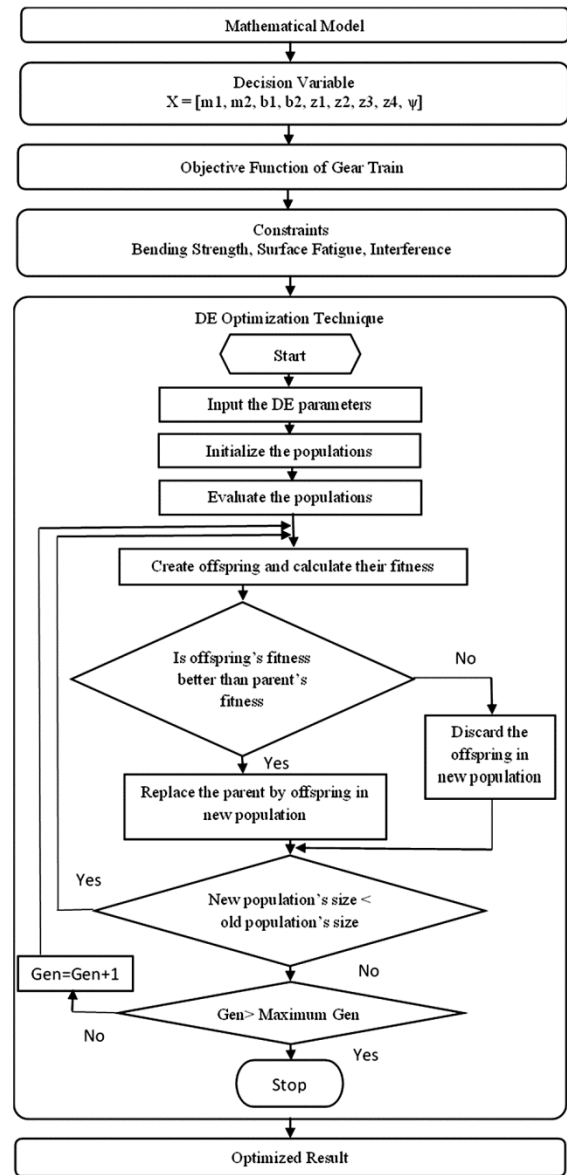


Fig. 2 — Flow chart of proposed methodology

train’s highest load carrying potential. The two-stage helical train’s optimization model is formulated as follow:

The overall volume of the gear train (Ft) is

$$Ft = [(d_1^2 + d_2^2)b_1 + (d_3^2 + d_4^2)b_2 + D_1^2L_1 + D_2^2L_2 + D_3^2L_3] \dots (2)$$

**Constraints**

The minimum volume objective function presents a number of constraints. The restrictions are used to limit the objective function so that appropriate parameters can be included. The formulas of bending force, surface fatigue strength, and interference have

Table 1 — Simulation input parameters

Parameters	Value (SI unit)	Parameters	Value (SI unit)
Transferred power, P	5.5 KW	Total gear ratio (i)	10
Input speed, $n_1$	1440 rpm	Material (Cementite steel)	16MnCr05
Output speed	144 rpm	Manufacturing process (Grade 6)	Fine
Work usage	Electricity motor	Brinell hardness, BHN	1460 N/mm <sup>2</sup>
Ultimate tensile strength, $S_u$	1100 N/mm <sup>2</sup>	Mean stress factor ( $K_{ms}$ )	1.4
Life Factor ( $C_{Li}$ )	1.0	Elastic coefficient ( $C_p$ )	$191\sqrt{MPa}$
Geometry factor ( $J_1, J_2$ )	0.5	Standard R.R moore endurance Limit $S'_n$	$0.5 \times S_u$
Service factor ( $C_s$ )	1.5	Deformation factor (C)	11400 N/mm <sup>2</sup>
Pressure angle ( $\theta$ )	20°	Mounting factor ( $K_m$ )	1.6
Overload factor ( $K_0$ )	1.0	Load factor ( $C_L$ )	1.0
Gradient factor ( $C_G$ )	1.0	Surface factor ( $C_s$ )	0.68
Reliability factor ( $K_r$ )	0.814	Temperature factor ( $K_t$ )	1.0
Reliability factor for $S_{fc}$ ( $C_r$ )	1.0	Surface fatigue strength, ( $S_{fc}$ )	$28 \times \text{BHN} - 69$
Helix angle ( $\Psi$ )	18 to 25	—	—

taken to form constraints for the designing of helical gears by adding the essential factors and their appropriate values are taken from reference.<sup>20,21</sup>

The bending strength for each gear pairs<sup>20,21</sup> are

$$g(1) = \frac{p_{t,1}}{b_1 j_1 m_1} (k_{v1} k_0 \times 0.93 k_m) - S_n^1 C_L C_G C_S K_r K_t K_{ms} \leq 0 \quad \dots (3)$$

$$g(2) = \frac{p_{t,2}}{b_2 j_2 m_2} (k_{v2} k_0 \times 0.93 k_m) - S_n^1 C_L C_G C_S K_r K_t K_{ms} \leq 0 \quad \dots (4)$$

where,  $k_{v1}$ ,  $k_{v2}$  are velocity factor for helical gear<sup>20</sup> and are given by

$$k_{v1} = \frac{5.6}{5.6 + \sqrt{v_1}}, k_{v2} = \frac{5.6}{5.6 + \sqrt{v_2}}$$

The surface damage that can occur to gear teeth is very diverse. Gear teeth undergo Hertz contact stresses, just as rolling aspect bearings, and elasto-hydrodynamic lubrication is commonly utilized. Abrasion, scoring, and pitting can occur in several combinations due to over-loading and lubrication degradation. It became evident that the durability of gear-teeth surfaces is a more complex topic than the capacity to sustain gear-teeth bending strain. Upon considering of all the parameters, the components of the surface fatigue constraint may be formulated as<sup>21</sup>

$$g(3) = C_p \sqrt{\frac{p_{t,1}}{b_1 d_1 I_1} \times \frac{\cos \Psi}{0.95 CR_1} (k_{v1} k_0 \times 0.93 k_m)} - S_{fc} C_{Li} C_r \leq 0 \quad \dots (5)$$

$$g(4) = C_p \sqrt{\frac{p_{t,2}}{b_2 d_3 I_2} \times \frac{\cos \Psi}{0.95 CR_2} (k_{v2} k_0 \times 0.93 k_m)} - S_{fc} C_{Li} C_r \leq 0 \quad \dots (6)$$

where,  $I_1, I_2$  are dimensionless constant<sup>19</sup> and are given by

$$I_1 = \frac{(1 + 0.00682 \times \theta) z_2}{4.0584 \times (z_2 + z_1)}, I_2 = \frac{(1 + 0.00682 \times \theta) z_4}{4.0584 \times (z_3 + z_4)}$$

$CR_1, CR_2$  are contact ratio for 1<sup>st</sup> stage and 2<sup>nd</sup> stage and are formulated as

$$CR_1 = \frac{\sqrt{rA_1^2 - rB_1^2} + \sqrt{rA_2^2 - rB_2^2} - c_1 \times \sin \theta}{\pi m_1 \cos \theta} + \frac{b_1 \times \sin \Psi}{\pi m_1},$$

$$CR_2 = \frac{\sqrt{rA_3^2 - rB_3^2} + \sqrt{rA_4^2 - rB_4^2} - c_2 \times \sin \theta}{\pi m_2 \cos \theta} + \frac{b_2 \times \sin \Psi}{\pi m_2}$$

And,  $c_1, c_2$  are center distance between gear 1, 2 and gear 3, 4 respectively<sup>21</sup> and are calculated by

$$c_1 = \frac{m_1(z_1 + z_2)}{2 \cos \Psi}, c_2 = \frac{m_2(z_3 + z_4)}{2 \cos \Psi}$$

Interference is a crucial consideration when designing the gear. The gear is where interference most likely occurs. Therefore, interference must be maintained in the optimization problem of gear. To eliminate interference, the following constraints will have to be convinced.<sup>21</sup>

$$g(5) = rA_1 - \sqrt{rB_1^2 + c_1^2 (\sin \theta)^2} \leq 0 \quad \dots (7)$$

$$g(6) = rA_3 - \sqrt{rB_3^2 + c_2^2 (\sin \theta)^2} \leq 0 \quad \dots (8)$$

$$g(7) = \frac{2}{(\sin \theta)^2} - z_1 \leq 0 \quad \dots (9)$$

$$g(8) = \frac{2}{(\sin \theta)^2} - z_2 \leq 0 \quad \dots (10)$$

$$g(9) = \frac{2}{(\sin \theta)^2} - z_3 \leq 0 \quad \dots (11)$$

$$g(10) = \frac{z}{(\sin \theta)^2} - z_4 \leq 0 \quad \dots (12)$$

$$g(11) = 77 - z_4 \leq 0 \quad \dots (13)$$

$$g(12) = z_4 - 110 \leq 0 \quad \dots (14)$$

where,  $rA_1, rA_2, rA_3$  and  $rA_4$  are addendum radius of gear 1, 2, 3 and 4 respectively which are given by

$$rA_1 = \frac{m_1}{2} \left( \frac{z_1}{\cos \psi} + 2 \right), rA_2 = \frac{m_1}{2} \left( \frac{z_2}{\cos \psi} + 2 \right)$$

$$rA_3 = \frac{m_2}{2} \left( \frac{z_3}{\cos \psi} + 2 \right), rA_4 = \frac{m_2}{2} \left( \frac{z_4}{\cos \psi} + 2 \right)$$

and  $rB_1, rB_2, rB_3, rB_4$  are base circle radius of gear 1, 2, 3 and 4, respectively which are given by

$$rB_1 = \frac{m_1 z_1 \cos \theta}{2 \cos \psi}, rB_2 = \frac{m_1 z_2 \cos \theta}{2 \cos \psi}$$

$$rB_3 = \frac{m_2 z_3 \cos \theta}{2 \cos \psi}, rB_4 = \frac{m_2 z_4 \cos \theta}{2 \cos \psi}$$

**Penalty Function**

The penalty function method is the one that is most frequently utilized among the several approaches for processing constraint functions. In this work, the objective function is modified as follows using the penalty function method<sup>11</sup> to handle constraint conditions:

$$Ft = [(d_1^2 + d_2^2)b_1 + (d_3^2 + d_4^2)b_2 + D_1^2 L_1 + D_2^2 L_2 + D_3^2 L_3] + PF \quad \dots (15)$$

where,  $PF = 5 \times 10^N \times \sum_{i=1}^{12} [\max(0, g(i))]^2$  known as penalty function<sup>18</sup> and  $N$  is punish factor coefficient and its value often greater than 2. After changing  $N$ 's value numerous times, the study sets  $N$  to 7, ensuring that there is essentially no volatility in the values of the objective function. If the inequality is preserved,  $g(i) \leq 0$  and  $\max(0, g(i))$  will be zero and hence  $PF$  will be zero. As a result, the constraint does not affect the  $Ft$ . And if the constraint is violated that means  $g(i) > 0$  and  $\max(0, g(i))$  should be greater than zero and a big term will be added to  $Ft$  function such that the solution is pushed back towards to the feasible area.

So, objective function of two stage helical gear train is expressed below

$$Ft = \begin{cases} [(d_1^2 + d_2^2)b_1 + (d_3^2 + d_4^2)b_2 + D_1^2 L_1 + D_2^2 L_2 + D_3^2 L_3] & \text{If constraint satisfied} \\ [(d_1^2 + d_2^2)b_1 + (d_3^2 + d_4^2)b_2 + D_1^2 L_1 + D_2^2 L_2 + D_3^2 L_3] + PF & \text{If constraint violated} \end{cases} \quad \dots (16)$$

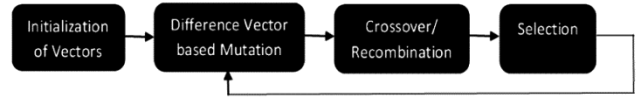


Fig. 3 — Main stages of DE algorithm

**Differential Evolution Algorithm**

One of the most reliable stochastic real-parameter optimization methods currently in use is Differential Evolution (DE). Over a decade ago, the DE algorithm first appeared as a very competitive evolutionary computing method.<sup>22</sup> It can handle multimodal, non-differentiable, and nonlinear objective functions. Neural networks with constrained and real integer weights have been trained using DE. DE is some of the most popular and effective software in the optimization field due to its ease of implementation and simplicity, fewer required parameters, excellent efficiency, and low area complexity. The DE algorithm's specifics are provided in literatures.<sup>23-25</sup>

In a  $D$ -dimensional real parameter space, DE seeks a world's most efficient point. As seen in Fig. 3, it actually operates through a simple cycle of steps. The algorithm begins by creating an initial population of  $NP$   $D$ -dimensional parameter vectors with actual values.

The parameters' values are restricted to a specific range, and the vectors are randomly started within that range. The target vectors are the parent vectors of the current generation. By scaling the difference between two randomly selected vectors from the population and combining the results with a third randomly selected vector also chosen from the same population, they are altered to produce donor vectors. The trial vectors are then produced using exponential or binomial crossover methods. Based on the values of the objective function that should be minimized, a resolution between the target and trial vector populations for the following generation may be made at the end. For complete DE equations Deng *et al.*<sup>26</sup> and Das & Suganthan<sup>27</sup> could be consulted.

**Result and Discussion**

Computer programming was necessary for the equation's optimization. Each and every parameter of the mathematical equation, i.e., Eq. 16, and constraints, i.e., Eqs. 3-14, are translated into code language for equation optimization. The optimization is carried out with the DE algorithm, and the coding is validated using a Matlab application and unique two-stage gear train's parameters. The technique performs a number of new operations to obtain the optimized

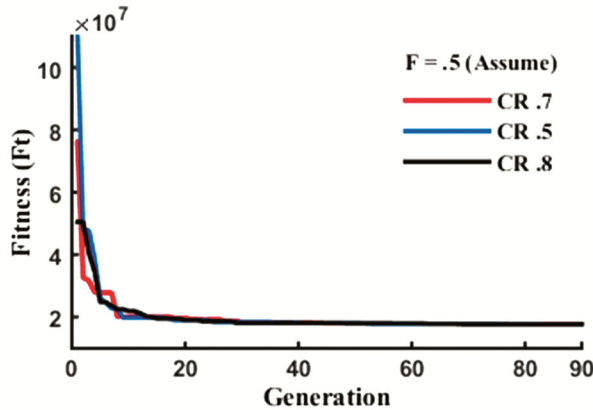


Fig. 4 — Variation of Ft vs. Generation at F=0.5 (Assume)

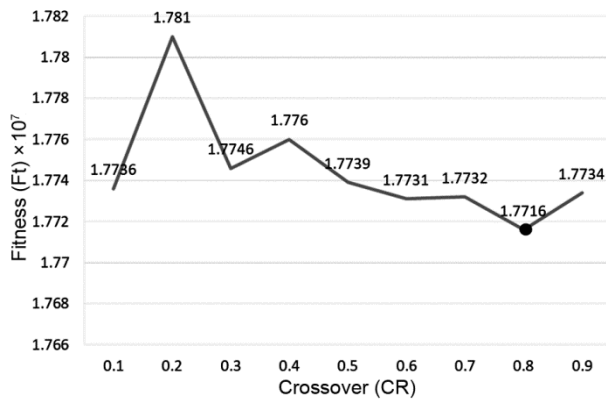


Fig. 5 — Variation of Ft vs. CR

minimum value and applies constraints optimization on nonlinear multi-variable problems with appropriate efficiency on a wide range of issues. The face width, number of teeth, and helix angle are chosen as the strongest design elements for optimizing the overall volume of the gear train. The ranges of the design variables are taken from the manufacturer’s catalogue, and the ranges for  $z_1, z_2, b_1, m_1, z_3, z_4, b_2, m_2, \Psi$  are taken as 14–20, 44–65, 60–80, 4–12, 14–20, 77–110, 85–105, 3–10, 18–25, respectively.

The input parameters are taken as initial population size = 90, number of generations = 90, Crossover varies from (0–1), and Mutation factor (F) value varies from (0–2). Therefore, to optimize the value of parameters, first keep the value of F constant i.e. (assume 0.5) then vary crossover (0–1) at interval of 0.1. After that DE is run with mentioned number of generation and it is observed that the objective function curve does not alter after 40 generation (Fig. 4).

As a result, generation number 90 is acknowledged as compromised one for better optimized value. After 90 generation take the value of crossover (CR) which

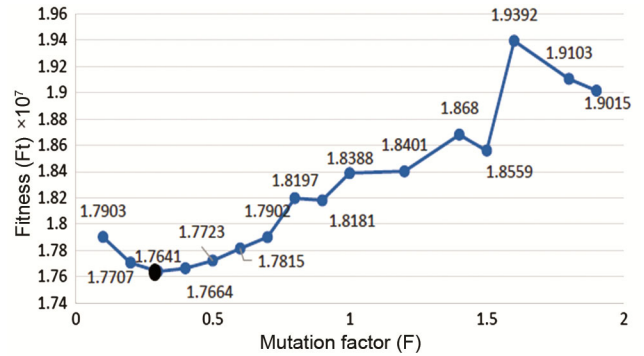


Fig. 6 — Variation of Ft vs. F

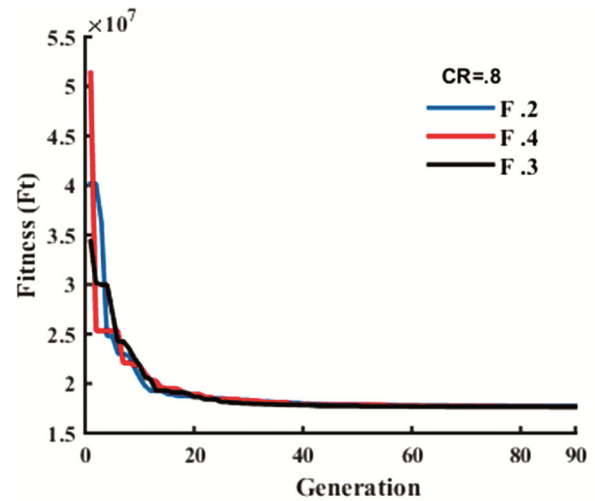


Fig.7 — Variation of Ft vs. Generation at CR=0.8

have optimized result. The variation of CR with fitness (Ft) is presented in Fig. 5. It is evident from Figs 4 and 5 that the optimum value of CR is 0.8. At CR = 0.8, the same process is repeat for F which varies from (0–2) at interval of 0.1. And it is seen from Fig. 6 and Fig. 7 that at F = 0.3, the optimum value of Ft is obtained. So, the optimized fitness curve with optimal DE parameters i.e., Mutation factor (F) = 0.3, and Crossover = 0.8 is presented in Fig. 8. Additionally, the corresponding optimized design parameters are provided in Table 2.

**Comparison of Results**

In this work, it is visible that the result remains consistent with population size = 90 and number of generations = 90. As a result, Table 3 displays 10 optimized good results for this population size and generation. Along with this, the outcomes of proposed technique are compared with the technique developed by Senthilkumar & Annamalai<sup>9</sup> to show the effectiveness of the proposed work. Comparative

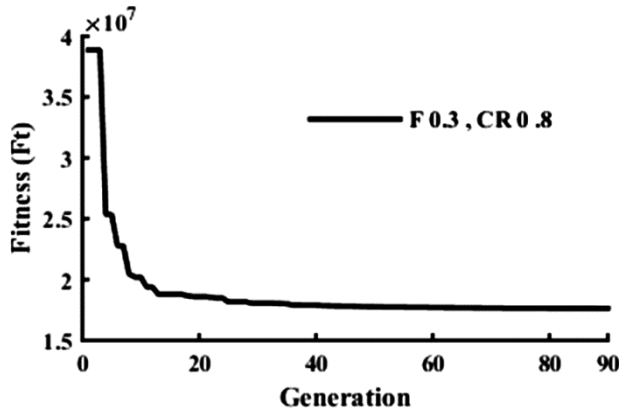


Fig. 8 — Optimized fitness curve

Table 2 — Results of DE optimized design parameters

Variable and objective	DE (Round off)
Teeth on gear 1	18
Teeth on gear 2	44
Module of gear 1 (mm)	4
Face width of gear 1 (mm)	60
Teeth on gear 3	19
Teeth on gear 4	78
Module of gear 3 (mm)	3
Face width of gear 3 (mm)	85
Volume, mm <sup>3</sup>	1.76 × 10 <sup>7</sup>
LCC (N)	8.89 × 10 <sup>3</sup>

Table 3 — DE for population size of 90 and 90 generation

Sl No.	m <sub>1</sub> (mm)	b <sub>1</sub> (mm)	z <sub>1</sub>	z <sub>2</sub>	m <sub>2</sub> (mm)	b <sub>2</sub> (mm)	z <sub>3</sub>	z <sub>4</sub>	Ψ	Volume × 10 <sup>7</sup> mm <sup>3</sup>	LCC × 10 <sup>3</sup> N
1	4	60	18	44	3	85	19	78	18.02	1.7634	8.897
2	4	60	18	44	3	85	19	78	18.05	1.7647	8.897
3	4	60	18	44	3	85	19	78	18.03	1.7635	8.899
4	4	60	18	44	3	85	20	77	18.00	1.7644	8.778
5	4	60	18	44	3	85	19	78	18.29	1.7636	8.899
6	4	60	18	44	3	85	18	78	18.00	1.7636	8.980
7	4	60	18	44	3	85	19	78	18.00	1.7640	8.901
8	4	60	18	44	3	85	20	77	18.03	1.7641	8.776
9	4	60	18	44	3	85	18	78	18.01	1.7645	8.980
10	4	60	18	44	3	85	19	78	18.03	1.7636	8.899

Table 4 — Comparison of result

Variable and objective	Catalog Value	GA <sup>9</sup> (Round off)	FS <sup>9</sup> (Round off)	DE (Round off)
Teeth on gear 1	14	18	19	18
Teeth on gear 2	44	57	60	44
Module, gear 1 (mm)	7	4	4	4
Face width of gear 1 (mm)	70	60	62	60
Teeth on gear 3	14	18	19	19
Teeth on gear 4	77	100	100	78
Module, gear 3 (mm)	3.5	3	3	3
Face width of gear 3 (mm)	95	85	86	85
Volume, mm <sup>3</sup>	2.29 × 10 <sup>7</sup>	2.16 × 10 <sup>7</sup>	2.15 × 10 <sup>7</sup>	1.76 × 10 <sup>7</sup>
LCC (N)	3.40 × 10 <sup>4</sup>	3.28 × 10 <sup>4</sup>	3.42 × 10 <sup>4</sup>	8.89 × 10 <sup>3</sup>

results show that the DE result is much better than the result reported by Senthilkumar & Annamalai.<sup>9</sup> The result obtained by DE algorithm by maintaining the gear ratio 10 and satisfying all the constraint give significant improvement over the result given by Senthilkumar & Annamalai<sup>9</sup> using Genetic Algorithm (GA) and Fminsearch Solver (FS) of Matlab. Unlike GA, DE algorithm does not need complex encoding and decoding process and unique genetic operator. The comparative results between proposed one and with the technique developed by Senthilkumar & Annamalai<sup>9</sup> i.e., with GA and FS are displayed in Table 4.

**Comparison with GA<sup>9</sup> Result**

When compared to the GA result provided by Senthilkumar & Annamalai<sup>9</sup>, it is found that a proper combination of design parameters and constraint variables results in an overall volume reduction of 18.51%. The number of teeth on gear 2 is reduced from 57 to 44, on gear 4 from 100 to 78, and on pinion 3 from 18 to 19, while the rest of the design variables remain constant. And there is a devastating change in the load carrying potential of the shaft because in this paper the gear ratio is maintained at 10, which is not maintained in the Senthilkumar & Annamalai<sup>9</sup> work (see Table 4).

Table 5 — Volume reduction comparison with respect to DE

Optimization techniques	Volume, mm <sup>3</sup>	% Reduction
GA <sup>9</sup>	$2.16 \times 10^7$	18.51
FS <sup>9</sup>	$3.28 \times 10^7$	18.14

### Comparison with FS<sup>9</sup> Result

When compared to the FS result provided by Senthilkumar & Annamalai<sup>9</sup>, it is found that a proper combination of design parameters and constraint variables results in an overall volume reduction of 18.14%. Here the no. of teeth on gear 2 is changed from 60 to 44, on gear 4 from 100 to 78, and on pinion 1 from 19 to 18. The first meshing teeth's face width is reduced from 62 to 60, while the second meshing teeth's face width is reduced from 86 to 85, and the rest of the design variables remain unchanged. And there is a devastating change in the load carrying potential of the shaft because in this paper the gear ratio is maintained at 10, which is not maintained in the reference paper given by Senthilkumar & Annamalai<sup>9</sup> work (see Table 4).

In general, the proposed approach shows an overall volume reduction of approximately 18% compared to other published optimization techniques, as shown in Table 5. This reduction leads to a reduction in the weight and size of the gear drive.

### Conclusions

The two-stage helical gear train's design parameters were optimized in this study using the DE optimization approach with a dynamic penalty function by minimizing the objective function i.e., the gear train's overall geometrical volume (size). The comparative results clearly illustrated that the DE results were superior to the reference paper results. The proposed approach showed a significant reduction i.e., 18.51% with GA and 18.14% with FS in the overall geometrical volume (size) of the two-stage helical gear train as compared to the published work. However, this work does not consider profile shift coefficient and specific sliding velocity and this may be considered as a future work for improving the design optimization problem. Further, this approach may be extended to multi-objective optimization problem for others multi-stage gear drive train. Other applications, such as minimizing the weight of the spring and the pulley system, can be handled using a similar strategy.

### References

- 1 Naveen P N E, Mayee M C, Gayathri P, Goriparthi B K & Reddy K R R M, Design and optimization of nylon 66 reinforced composite gears using genetic algorithm, *Mater Today Proc*, **46** (2021) 514–519.
- 2 Xin W, Zhang Y, Fu Y, Yang W & Zheng H, A multi-objective optimization design approach of large mining planetary gear reducer, *Sci Rep*, **13**(1) (2023) 18640.
- 3 Dixit Y & Kulkarni M S, Multi-objective optimization with solution ranking for design of spur gear pair considering multiple failure modes, *Tribol Int*, **180** (2023) 108284.
- 4 Marafona J D, Carneiro G N, Marques P M, Martins R C, António C C & Seabra J H, Gear design optimization: Stiffness versus dynamics, *Mech Mach Theory*, **191** (2024) 105503.
- 5 Patil M, Ramkumar P & Shankar K, Multi-objective optimization of the two-stage helical gearbox with tribological constraints, *Mech Mach Theory*, **138** (2019) 38–57.
- 6 Hoseiniasl M & Fesharaki J J, 3D optimization of gear train layout using particle swarm optimization algorithm, *J Appl Computat Mech*, **6**(4) (2020) 823–840.
- 7 Le X H & Vu N P, Multi-objective optimization of a two-stage helical gearbox using taguchi method and grey relational analysis, *Appl Sci*, **13**(13) (2023) 7601.
- 8 Savsani V, Rao R & Vakharia D, Optimal weight design of a gear train using particle swarm optimization and simulated annealing algorithms, *Mech Mach Theory*, **45**(3) (2010) 531–541.
- 9 Senthilkumar R & Annamalai K, Multi-objective optimization of two-stage helical gear train, *ARPN J Eng Appl Sci*, **11**(16) (2016) 10103–10109.
- 10 Sanghvi R, Vashi A, Patolia H & Jivani R, Multiobjective optimization of two-stage helical gear train using NSGA-II, *J Optim*, **2014** (2014).
- 11 Song X & Cao X, The optimization design of two stages helical gear reducer based on ga and nastran, in *First Int Conf Informat Sci Machin Mater Energy* (Atlantis Press) 2015, 1461–1466.
- 12 Huang H Z, Tian Z G & Zuo M J, Multiobjective optimization of three-stage spur gear reduction units using interactive physical programming, *J Mech Sci Technol*, **19**(5) (2005) 1080–1086.
- 13 Yokota T, Taguchi T & Gen M, A solution method for optimal weight design problem of the gear using genetic algorithms, *Comput Ind Eng*, **35**(3–4) (1998) 523–526.
- 14 Thompson D F, Gupta S & Shukla A, Tradeoff analysis in minimum volume design of multi-stage spur gear reduction units, *Mech Mach Theory*, **35**(5) (2000) 609–627.
- 15 Golabi S, Fesharaki J J & Yazdipoor M, Gear train optimization based on minimum volume/weight design, *Mech Mach Theory*, **73** (2014) 197–217.
- 16 Mendi F, Baskal T, Boran K & Boran F E, Optimization of module, shaft diameter and rolling bearing for spur gear through genetic algorithm, *Expert Syst Appl*, **37** (2010) 8058–8064.
- 17 Padmanabhan S, Chandrasekaran M & Srinivasa V, Design optimization of worm Gear drive, *Int J Mining Metall Mech Eng*, **1** (2013) 57–61.
- 18 Tong B S & Walton D, The optimisation of internal gears, *Int J Mach Tools Manuf*, **27**(4) (1987) 491–504.
- 19 Maitra G M, *Handbook of Gear Design*, (Tata McGraw-Hill Education) 1994.
- 20 Bhandari V, *Design of Machine Elements*, (Tata McGraw-Hill Education) 2010.

- 21 Juvinall R C & Marshek K M, *Fundamentals of Machine Component Design* (John Wiley & Sons) 2020.
- 22 Storn R & Price K, Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces, *J Glob Optim*, **11(4)** (1997) 341–359.
- 23 Sun C, Zhou H & Chen L, Improved differential evolution algorithms, in *Proc IEEE Int Conf Comput Sci Automatic Engg*, **3** (2012) 142–145.
- 24 Yang L G & Guang L M, The summary of differential evolution algorithm and its improvements, in *Proc IEEE Int Conf Adv Comput Theory Eng*, **3** (2010) 153–156.
- 25 Ahadzadeh B & Menhaj M B, A modified differential evolution algorithm based on a new mutation strategy and chaos local search for optimization problems, in *Proc IEEE Int Conf Comput Knowl Engg*, (2014) 468–473.
- 26 Deng W, Shang S, Cai X, Zhao H, Song Y & Xu J, An improved differential evolution algorithm and its application in optimization problem, *Soft Comput*, **25(7)** (2021) 5277–5298.
- 27 Das S & Suganthan P N, Differential evolution: A survey of the state-of-the-art, *IEEE Trans Evol Comput*, **15(1)** (2010) 4–31.