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Reynolds Equation Development for Double Porous Layered Circular Squeeze Film Bearing using Shliomis Model

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This study presents theoretical development of Reynolds type equation for the double porous layered circular squeeze film bearing lubricated with ferro-fluid considering Shliomis flow Model. The external magnetic field is considered variable and oblique due to its benefit of generating utmost field at the essential contact region of the bearing. The effects of slip and squeeze velocities are also taken in to account. The squeeze film circular bearings are made up of circular impermeable discs having porous layer applied on both upper disc and lower disc. It is assumed that inside the porous matrix, the Darcy's Law is valid. The Shliomis flow model is considered as it includes the rotation effects of magnetic particles and carrier liquid. In addition, the porosity effect is also included due to its beneficial assets of self-lubrication. The main objective of the study is to develop the Reynolds Equation considering Shliomis model theoretically. By using the basic assumptions of the lubrication theory, the Reynolds type Equation is finally obtained which contains the pressure and it can be obtained by solving Reynolds Equation with suitable pressure boundary conditions.

Keywords: Circular bearing, Ferro-fluid, Porous matrix, Reynolds equation, Shliomis model

Introduction

A Ferro-Fluid (FF) is a fluid which attracts the magnetic particles, created by suspending very small magnetic (ferromagnetic) particles to carrier liquid. Neuringer-Rosensweig[1] introduced ferro-fluid flow model that considered only magnetic force lacking the rotation effects of magnetic particles and carrier liquid. Ferro-fluids are invented by Neuringer-Rosensweig[1] and many researchers studied its applications from different viewpoints as lubricant for different bearing systems.[2–5] In all the studied problems, it is observed that by using the FF as a lubricant, performances of the systems improved as related to the conventional fluids. Prajapati[6] considered the effect of FF on various porous (squeeze) film bearing systems such as conical, circular, elliptic, annular etc. In his conclusions, he established that the load capacity improves with the rise in magnetization factor. Montazeri[7] in his numerical analysis of FF lubricated journal bearings found that as compared to the conventional fluid as lubricant, FF gives better load capacity. Similar conclusions have also been made for different bearing systems by Patel *et al.*[8–10] in their study.

Prakash & Vij[11] considered a porous slider with conventional fluid and concluded that friction and load capacity reduced while the friction coefficient is increased due to porosity. In the very next year, Murti[12] studied the squeeze behaviour in (porous) circular disks. Subsequently, Kulkarni and Kumar[13] proposed a new equation for porous lubricated bearings, accounting for non-isotropic permeability and the non-adherence of the lubricating fluid at the porous interface. Wu[14] later provided a review of porous squeeze films. Gupta and Bhat[15] investigated inclined (porous) slider bearings and concluded that the application of a transverse magnetic field to the bearing system, along with a steering lubricant, increases both load capacity and friction. Prakash & Tiwari[16] considered various roughness designs and exposed that surface roughness substantially impacts on the functioning mechanism of porous bearings.

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Due to magnetic particles as well as carrier liquid, dissimilar (angular) velocities of rotations, frictional forces arise. Such forces result in rise in real viscosity of the FF and it has key influence on pressure. The rotations of magnetic particles and carrier liquid in the Ferro-fluid flow model were taken into account by Shliomis[17] under applied magnetic force. Several authors[18–22] studied Shliomis model-based lubrication bearing problems with different viewpoints. Shukla & Kumar[18] considered slider bearing lubricated by FF by means of constant transverse magnetic force by ignoring the relaxation time of rotations of particles. They have developed pressure equation considering that the FF is saturated and magnetic moment relaxation time is almost negligible. Shah and Bhat[19] investigated FF-lubricated squeeze films among curved annular plates and derived the pressure equation without the assumptions made by Shukla and Kumar.[18] They demonstrated that both the load capacity and the approaching time (of squeeze film) can be enhanced by increasing the solid-phase volume concentration in the FF as well as the intensity of the external magnetic force. Furthermore, Shah[20] extended the analysis done by Shah & Bhat[19] by including the upper plate rotation effect, and considered various shapes of the upper plate such as flat, secant and exponential. As a conclusion, he found that response time and load-carrying capacity increase as the increase of the upper plate rotation as well as particles volume fraction.

Patel & Deheri[21] studied FF lubricated squeeze film considering rotating (rough curved) circular discs having attached porous matrix. In their study, it was detected that roughness effect cannot be neglected while designing such bearing system; however appropriate magnetization remains significant. Shah & Shah[22] considered circular bearings with flexible magnetic field and discs rotations. For the first time, they noticed a replica of the secant shape. They found that highest load capacity is achieved just in case of exponential shape; on the contrary lowest load capacity is achieved in the case of secant bearing. Also, load capacity increases even when lower disc rotation is zero.

Singh & Gupta[23] considered curved slider (FF lubricated bearing) with influence of the transverse (magnetic) force and concluded that due to effect of magnetic particle's rotations, improvement is seen in damping capacities in addition to stiffness. Shah & Patel[24] considered double porous coated (axially undefined) journal bearing and found that the load capacity doubles when FF is considered as lubricant rather than ordinary lubricant. Shah & Patel[29,30] also studied circular porous squeeze film bearings and concluded that uniform magnetic force have no effect on the bearing performance. They have used Neuringer-Rosensweig flow model to derive the Reynolds type Equation. Results for the load-carrying capacity are calculated and then compared. Shah *et al*.[31] studied squeeze film bearing and concluded that squeeze velocity have no effects on pressure as well as load carrying capacity.

So far as per the reviewed literature, the authors are motivated to study double porous layered circular squeeze film bearing lubricated by FF considering Shliomis Model. This study derived the Reynolds type equation for the double porous layered circular squeeze film bearing lubricated with FF considering Shliomis flow Model. The model presented by Shliomis is vital because it considers the rotation effects of both the carrier liquid plus magnetic particles. It also acts in a different way when a changing magnetic field is involved.

This study is important, as from the Reynolds type equation for the considered problem, researchers can extend this work in many directions in future.

In this study, it is assumed that the Darcy's Law is valid in the (porous) area. In the film section as well as in the porous area, the continuity equation is utilized as well.

Formulation of the Problem

The bearing system considered for the present study is presented in Fig. 1. As shown in the diagram, it is

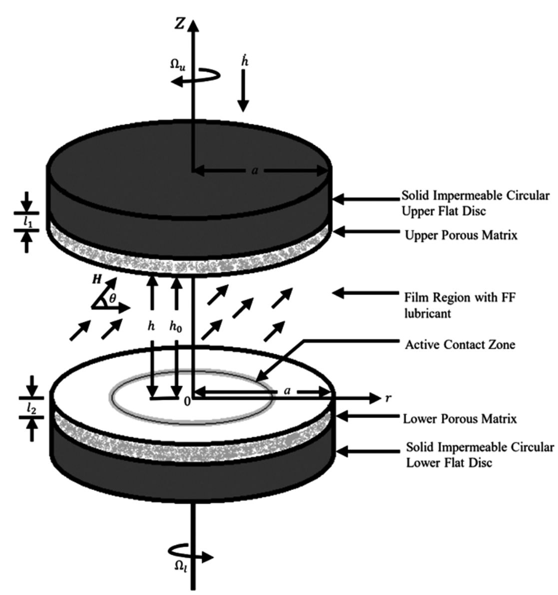

Fig. 1 — Double porous layered circular bearing

consists of two circular discs (upper and lower) of same radius 'a'. The upper plate could be of different shapes such as exponential, secant or flat, while the lower plate is flat. The porous matrix of uniform thicknesses l_1 and l_2 are assorted on these upper and lower discs respectively. Also, the upper disc moves towards the lower plate with some velocity (called the squeeze velocity) $\dot{h} = \frac{dh}{dt}$, where $h_0 = h$ represents the central film thickness and t denotes time. Additionally, both plates are rotated with their own respective angular (rotational) velocities Ω_u and Ω_l. The space between these two discs, referred to as the fluid film area, is filled with FF.

Shliomis Flow Model

Shliomis[17] discovered that magnetic particles of FFs are able to relax under changing applied magnetic force in two ways. One is by rotations of such particles within the fluid and other by magnetic moment rotation in such particles. Particle rotation is described through Brownian relaxation time τ_B and τ_s, the relaxation time parameter. If the flow is steady and neglecting inertial term and the second derivatives of $\bar{S}$, the internal angular momentum, the equations of the Shliomis Model[17] to govern the flow are,

$$-\nabla p + \eta \nabla^2 \bar{q} + \mu_0 (M.\nabla) H + \frac{1}{2\tau_s} \nabla \times (\bar{S} - I\bar{\Omega}) = 0 \quad \dots (1)$$

$$\bar{S} = I\bar{\Omega} + \mu_0 \tau_s (M \times H) \quad \dots (2)$$

$$M = M_0 \frac{H}{H} + \frac{\tau_B}{I} (\bar{S} \times M), \quad \dots (3)$$

Continuity Equation:

$$\nabla . \bar{q} = 0 \quad \dots (4)$$

Electro-magnetic field Equation:

$$\nabla \times H = 0 \quad \dots (5)$$

$$\nabla . (H + M) = 0 \quad \dots (6)$$

$$M = \bar{\mu} H \quad \dots (7)$$

where, I is the total inertia moment for the particles for each unit volume, $\bar{\Omega} = \frac{1}{2} \nabla \times \bar{q}$.

As per the discussions of Shliomis[17], Shukla & Kumar[18] and Shah & Bhat[19], combining Eq. (2) with Eqs. (1) and (3), one can obtain

$$-\nabla p + \eta \nabla^2 q + \mu_0 (M \cdot \nabla) H + \frac{\mu_0}{2} \nabla \times (M \times H) = 0 \quad \dots (8)$$

and

$$M = M_0 \frac{H}{H} + \tau_B (\bar{\Omega} \times M), \quad \dots (9)$$

By ignoring the terms of τ_B and τ_s, and by substituting M in Eq. (8), one obtains

$$-\nabla p + \left(\eta + \frac{\mu_0}{4} \tau_B M \cdot H\right) \nabla^2 q + \mu_0 (M \cdot \nabla) H + \frac{\mu_0 \tau_B}{2} \begin{bmatrix} \nabla(\bar{\Omega} \cdot H) \times M \\ +(\bar{\Omega} \cdot H) \nabla \times M - \nabla(M \cdot H) \times \bar{\Omega} \end{bmatrix} = 0 \quad \dots (10)$$

The first approximation of M as given by Eq. (9) is

$$M = M_0 \frac{H}{H}$$

To get the second approximation of M, substitute above value to Eq. (9)

$$M = M_0 \frac{H}{H} + \frac{M_0}{H} \bar{\tau} (\bar{\Omega} \times H), \quad \dots (11)$$

where,

$$\bar{\boldsymbol{\tau}} = \frac{\tau_B}{1 + \left(\mu_0 M_0 H \tau_B \tau_s / I\right)}$$

Analysis of the Problem

Assuming the velocity gradient across the film is dominant, u can be expressed as a linear function of z and the flow is axially symmetric, combining Eq. (1) and Eq. (2) results

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta\left(1 + \frac{\mu_0 M_0 H \bar{\tau}}{4\eta}\right)} \left[\frac{dp}{dr} - \mu_0 M_0 \frac{dH}{dr} - \rho r \left(\frac{z}{h} \Omega_r + \Omega_l\right)^2\right] \quad \dots (12)$$

where, $\Omega_r = \Omega_u - \Omega_l$ and u denotes the radial factor of $\boldsymbol{q}$. The inclination of θ of $\boldsymbol{H} = H(r)(cos\theta, 0, sin\theta), \theta = \theta(r, z)$ to radial direction is supposed to be tiny and may be determined from Eq. (5). To consider the active contact zone in the neighbourhood of $r = {}^{2a}/_{3}$, the expression of H (Shah & Kataria[25]) is

$$H = K r^2 (a - r) \quad \dots (13)$$

in which K is the parameter chosen such that the both sides dimensions of Eq. (13) become equal. For suspension of spherical particles, defining the quantities[17] listed below

$$M_0 = nm\left(coth\xi - \frac{1}{\xi}\right), \ \xi = \frac{\mu_0 m H}{k_B T}, \tau_B = \frac{3\eta V}{k_B T},$$
$$V = \frac{\varphi}{n}, \tau_s = \frac{1}{6\eta\varphi}, \quad \tau = \frac{3}{2}\varphi \frac{\xi - tanh\xi}{\xi + tanh\xi}, \quad \dots (14)$$

Eq. (12) becomes

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta(1+\tau)}\left[\frac{d}{dr}\left(p - nk_BTln\frac{sinh\xi}{\xi}\right) - \rho r\left(\frac{z}{h}\Omega_r + \Omega_l\right)^2\right], \quad \dots (15)$$

where, n, m, k_B, T, φ, ξ and τ respectively denotes No. of particles/unit volume, particle's magnetic moment, Boltzmann parameter, temperature, particle's volume concentration, Langevin's parameter which is nothing but the non-dimensional form of H and parameter of rotational viscosity.

Solving Eq. (15) under slip conditions[24,26]

$$u = \frac{1}{s_1}\frac{\partial u}{\partial z} \text{at} z = 0 \text{ and } u = -\frac{1}{s_2}\frac{\partial u}{\partial z} \text{ at } z = h, \quad \dots (16)$$

where, $\frac{1}{s_1} = \frac{\sqrt{\varphi_r \eta_r}}{5}$ and $\frac{1}{s_2} = \frac{\sqrt{\psi_r m_r}}{5}$ are slip parameters, η_r and m_r denotes the r-directional porosity, φ_r and ψ_r denotes permeabilities for lower and upper porous matrix respectively, one attains

$$u = \frac{1}{s\eta(1+\tau)}\left[\begin{matrix}\left(\frac{sz^2-(2s_1h+s_1s_2h^2)z-(2h+s_2h^2)}{2}\right)\frac{d}{dr}\left(p - nk_BTln\frac{sinh\xi}{\xi}\right) \\ -\rho r\left\{\begin{matrix}\left(\frac{sz^4-s_1h^3(4+s_2h)z-h^3(4+s_2h)}{12h^2}\right)\Omega_r^2 \\ +\left(\frac{sz^2-(2s_1h+s_1s_2h^2)z-(2h+s_2h^2)}{2}\right)\Omega_l^2 \\ +\left(\frac{sz^3-(3s_1h^2+s_1s_2h^3)z-(3h^2+s_2h^3)}{3h}\right)\Omega_r\Omega_l\end{matrix}\right\}\end{matrix}\right], \quad \dots (17)$$

where, $s = s_1 + s_2 + s_1s_2h$.

Referring Shah & Kataria[25], Shah & Bhat[27] and authenticity of the Darcy's Law, the velocity constituents $\overline{u_1}$ and $\overline{w_1}$ of the fluid velocity for the upper porous section in r-direction and z-direction respectively are

$$\overline{u_1} = -\frac{\psi_r}{\eta}\left[\frac{\partial}{\partial r}\left(P_1 - nk_BTln\frac{sinh\xi}{\xi}\right) - \rho r\Omega_u^2 - \frac{1}{4}\frac{\partial}{\partial z}\left(\mu_0 M_0 H\bar{\tau}H\frac{\partial u}{\partial z}\right)\right] \quad \dots (18)$$

and

$$\overline{w_1} = -\frac{\psi_z}{\eta}\left[\frac{\partial}{\partial z}\left(P_1 - nk_BTln\frac{sinh\xi}{\xi}\right) + \frac{1}{4r}\frac{\partial}{\partial r}\left(r\mu_0 M_0 H\bar{\tau}H\frac{\partial u}{\partial z}\right)\right] \quad \dots (19)$$

where, P_1 denotes the pressure of fluid in upper (porous) matrix. Similarly, the velocity constituents $\overline{u_1}$ and $\overline{w_1}$ of the fluid velocity for the lower porous section in r-direction and z-direction respectively are

$$\overline{u_2} = -\frac{\varphi_r}{\eta}\left[\frac{\partial}{\partial r}\left(P_2 - nk_BTln\frac{sinh\xi}{\xi}\right) - \rho r\Omega_l^2 - \frac{1}{4}\frac{\partial}{\partial z}\left(\mu_0 M_0 H\bar{\tau}H\frac{\partial u}{\partial z}\right)\right] \quad \dots (20)$$

and

$$\overline{w_2} = -\frac{\varphi_z}{\eta}\left[\frac{\partial}{\partial z}\left(P_2 - nk_BTln\frac{sinh\xi}{\xi}\right) + \frac{1}{4r}\frac{\partial}{\partial r}\left(r\mu_0 M_0 H\bar{\tau}H\frac{\partial u}{\partial z}\right)\right] \quad \dots (21)$$

where, P_2 denotes the pressure of fluid in lower (porous) matrix. The continuity Eq. (4) for fluid film area in the cylindrical (polar) coordinates is

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \quad \dots (22)$$

for the upper and lower porous sections become

$$\frac{1}{r}\frac{\partial}{\partial r}(r\overline{u_1}) + \frac{\partial \overline{w_1}}{\partial z} = 0 \quad \dots (23)$$

And

$$\frac{1}{r}\frac{\partial}{\partial r}(r\overline{u_2}) + \frac{\partial \overline{w_2}}{\partial z} = 0 \quad \dots (24)$$

respectively. Substituting Eq. (18) and Eq. (19) in Eq. (23) yields

$$\frac{\psi_r}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(P_1 - nk_BTln\frac{sinh\xi}{\xi}\right)\right] - 2\rho r\Omega_u^2\frac{\psi_r}{r} - \frac{\psi_r}{4r}\frac{\partial^2}{\partial r\partial z}\left(r\mu_0 M_0 H\frac{\partial u}{\partial z}\right) + \frac{\psi_z}{z}\frac{\partial^2}{\partial z^2}\left(P_1 - nk_BTln\frac{sinh\xi}{\xi}\right) + \frac{\psi_z}{4r}\frac{\partial^2}{\partial r\partial z}\left(r\mu_0 M_0 H\bar{\tau}H\frac{\partial u}{\partial z}\right) = 0 \quad \dots (25)$$

Integrating Eq. (25) forz with the limits of upper porous disc depth$(h, h + l_1)$ one obtains

$$\psi_z\frac{\partial}{\partial z}\left(P_1 - nk_BTln\frac{sinh\xi}{\xi}\right)\Big|_{z=h} = \frac{\psi_r l_1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial z}\left(P_1 - nk_BTln\frac{sinh\xi}{\xi}\right)\right] - \frac{(\psi_r-\psi_z)}{4r}\frac{\partial}{\partial r}\left(\frac{2r\mu_0 M_0 H\bar{\tau}l_1}{\eta(1+\tau)}\right)\frac{d}{dr}\left(p - nk_BTln\frac{sinh\xi}{\xi}\right) + \frac{(\psi_r-\psi_z)}{4r}\frac{\partial}{\partial r}\left(\frac{r\mu_0 M_0 H\bar{\tau}\rho r(\Omega_r+\Omega_l)^2}{\eta(1+\tau)}\right) - 2\rho\Omega_u^2\psi_r l_1 \quad \dots (26)$$

by Morgan-Cameron approximation[11] and using the fact that surface at $z = h + l_1$ is hard impermeable. Similarly, substituting Eq. (20) and Eq. (21) in Eq. (24) yields

$$\frac{\varphi_r}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(P_2 - nk_BTln\frac{sinh\xi}{\xi}\right)\right] - 2\rho\varphi_r\Omega_l^2 - \frac{\varphi_r}{4r}\frac{\partial^2}{\partial r\partial z}\left(r\mu_0 M_0\bar{\tau}H\frac{\partial u}{\partial z}\right) + \varphi_z\frac{\partial^2}{\partial z^2}\left(P_2 - nk_BTln\frac{sinh\xi}{\xi}\right) + \frac{\varphi_z}{4r}\frac{\partial^2}{\partial r\partial z}\left(r\mu_0 M_0 H\bar{\tau}H\frac{\partial u}{\partial z}\right) = 0 \quad \dots (27)$$

Integrating Eq. (27) for z with the limits of lower disc depth $(-l_2, 0)$ one may achieve

$$\varphi_z\frac{\partial}{\partial z}\left(P_2 - nk_BTln\frac{sinh\xi}{\xi}\right)\Big|_{z=0} = -\frac{\varphi_r l_2}{2}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(P_2 - \right.\right.$$

$$nk_BTln\frac{sinh\xi}{\xi}\Big)\Big] - \frac{(\varphi_r-\varphi_z)}{4r}\frac{\partial}{\partial r}\left(\frac{r\mu_0M_0H\bar{\tau}l_2}{\eta(1+\tau)}\right)\frac{d}{dr}\Big(p - nk_BTln\frac{sinh\xi}{\xi}\Big) - \frac{(\varphi_r-\varphi_z)}{4r}\frac{\partial}{\partial r}\left(\frac{r^2\mu_0M_0H\bar{\tau}l_2\rho\Omega_l^2}{\eta(1+\tau)}\right) + 2\rho\Omega_l^2\varphi_r l_2 \quad \dots (28)$$

by Morgan-Cameron approximation[11] and using the fact that surface at $z=-l_2$ is hard impermeable.

Supposing that the velocity constituents are continuous across film (porous) boundary, gives

$$w|_{z=h} = w_h = \dot{h} + \overline{w_1}|_{z=h}\,,\ w|_{z=0} = w_0 = \overline{w_2}|_{z=0}$$

and the integral form of the continuity Equation $\frac{1}{r}\frac{\partial}{\partial r}\int_0^h ru\,dz + w_h - w_0 = 0$, yields (by using Eqs. (17), (26) and (28))

$$\frac{1}{r}\frac{\partial}{\partial r}\Bigg[\frac{r}{s\eta(1+\tau)}\Bigg\{\frac{h^2}{12}\Big((2s-6s_1-6s_2)h - 3s_1s_2h^2 - 12\Big)\frac{d}{dr}\Big(p - nk_BTln\frac{sinh\xi}{\xi}\Big) + \frac{\rho r h^2}{120}\big(5s_1s_2h^2 + h(20s_1+10s_2-2s) + 40\big)\Omega_r^2 + \frac{\rho r h^2}{12}\big(3s_1s_2h^2 + h(6s_1+6s_2-2s) + 12\big)\Omega_l^2 + \frac{\rho r h^2}{12}\big(2s_1s_2h^2 + h(6s_1+4s_2-s) + 12\big)\Omega_r\Omega_l\Bigg\}\Bigg] + \dot{h}$$
$$-\frac{1}{\eta}\Big[(\psi_r l_1 + \varphi_r l_2)\frac{1}{r}\frac{\partial}{\partial r}\Big\{r\frac{\partial}{\partial r}\Big(p - nk_BTln\frac{sinh\xi}{\xi}\Big)\Big\} - \frac{1}{4r}(2\psi_r l_1 - 2\psi_z l_1 + \varphi_r l_2 - \varphi_z l_2)\frac{\partial}{\partial r}\left(\frac{r\mu_0M_0H\bar{\tau}}{\eta(1+\tau)}\right)\frac{d}{dr}\Big(p - nk_BTln\frac{sinh\xi}{\xi}\Big) + \frac{1}{4r}\{(\psi_r-\psi_z)\rho l_1(\Omega_r+\Omega_l)^2 + (\varphi_r-\varphi_z)l_2\rho\Omega_l^2\}\frac{\partial}{\partial r}\left(\frac{r^2\mu_0M_0H\bar{\tau}}{\eta(1+\tau)}\right) + 2\rho\big(\Omega_u^2\psi_r l_1 - \Omega_l^2\varphi_r l_2\big)\Big] - \frac{1}{4\eta r}\Big\{\psi_z\frac{\partial}{\partial r}\Big(r\mu_0M_0\bar{\tau}H\frac{\partial u}{\partial z}\Big)\Big|_{z=h} - \varphi_z\frac{\partial}{\partial r}\Big(r\mu_0M_0\bar{\tau}H\frac{\partial u}{\partial z}\Big)\Big|_{z=0}\Big\} = 0, \quad \dots (29)$$

in which P_1 and P_2 represents fluid pressure in upper and lower porous matrix respectively are considered same as p – fluid pressure in film region.

Using Eqs. (19), (21), (26) and (28) in above Eq. (29) one obtains the Reynolds type Equation for the considered problem as

$$\frac{1}{r}\frac{\partial}{\partial r}\Big[G\frac{d}{dr}\Big(p - nk_BTln\frac{sinh\xi}{\xi}\Big) + F\Big] = E \quad \dots (30)$$

where,

$$G = \frac{r}{s\eta(1+\tau)}\Big\{\frac{h^2}{12}\Big((2s-6s_1-6s_2)h - 3s_1s_2h^2 - 12\Big)\Big\} - \frac{1}{\eta}(\psi_r l_1 + \varphi_r l_2)r + \frac{r\mu_0M_0H\bar{\tau}}{4\eta(1+\tau)}\Big[\Big\{\frac{1}{\eta}\big(2(\psi_r-\psi_z)l_1 + (\varphi_r-\varphi_z)l_2\big)\Big\} + \frac{\psi_z}{s}\Big(\frac{2sh - 2s_1h + s_1s_2h^2}{2}\Big) + \frac{\varphi_z}{s}\Big(\frac{2s_1h + s_1s_2h^2}{2}\Big)\Big],$$

$$F = \frac{\rho r h^2}{120}\{(5s_1s_2h^2 + h(20s_1+10s_2-2s) + 40)\Omega_u^2 + (25s_1s_2h^2 + h(80s_1+50s_2-12s) + 160)\Omega_r\Omega_l + (15s_1s_2h^2 + h(20s_1+110s_2-12s) + 280)\Omega_l^2\} + \frac{\rho}{4}\{(\psi_r-\psi_z)l_1\Omega_u^2 + (\varphi_r-\varphi_z)l_2\Omega_l^2\}\frac{r^2\mu_0M_0H\bar{\tau}}{\eta(1+\tau)} - \frac{\psi_z\rho r}{4\eta^2 s(1+\tau)}\Big[\Big(\frac{4hs_1+s_1s_2h^2}{12}\Big)\Omega_u^2 + \Big(\frac{-2hs_1+s_1s_2h^2}{12}\Big)\Omega_l^2 + \Big(\frac{5hs_1-2s_1s_2h^2}{6}\Big)\Omega_u\Omega_l\Big] + \frac{\varphi_z\rho r}{4s\eta(1+\tau)}\Big[\Big(\frac{4hs_1+s_1s_2h^2}{12}\Big)\Omega_u^2 + \Big(\frac{4hs_1+11s_1s_2h^2}{12}\Big)\Omega_l^2 + \Big(\frac{10hs_1+3s_1s_2h^2}{6}\Big)\Omega_u\Omega_l\Big],$$

$$E = -\dot{h} - \frac{2\rho}{\eta}\big(\Omega_u^2\psi_r l_1 - \Omega_l^2\varphi_r l_2\big).$$

Future scopes

Some of the future scopes are listed below:

1. Using the Reynolds type Equation and introducing dimensionless quantities, an expression of dimensionless load-carrying capacity can be derived. Solving this expression under pressure boundary conditions; one may obtain pressure expression for the considered problem.
2. In this problem one may consider various forms of the lower and upper disc, by considering the film thickness expression h[22] as:
   a) For exponentially curved disc
   $$h = h_0e^{-\beta r^2}, 0 \le r \le a\,,$$
   b) For secant curved disc
   $$h = h_0sec(\beta r^2), 0 \le r \le a\,,$$
   c) For mirror image of secant curved disc
   $$h = 2h_0 - h_0sec(\beta r^2), 0 \le r \le a\,,$$
   d) For parallel disc
   $$h = h_0\,, 0 \le r \le a,$$
   where, β and $r$denotes (from a) to d)) the curvature and radial coordinate respectively and h_0 is the film thickness at centre.
3. Moreover, for the performance viewpoint, dimensionless load capacity can be calculated and analyze for variation of different parameters like as porous thickness, slip velocity, squeeze velocity, permeability and eccentricity.
4. As a comparison view point, comparison between three well known flow models named Neuringer- Rosensweig, Jenkins as well as Shliomis model can be done and one can validate the obtained results with the available literature. Also, based on obtained results, which model gives better performance for the present problem can be suggested.

Conclusions

For double porous layered circular squeeze film bearing lubricated with ferro-fluid considering Shliomis Model, the Reynolds type Equation is derived theoretically by using the usual assumptions of lubrication theory. This Equation is significant to enhance this study further in many directions which are listed in future scopes. However, from this article, one may motivate to analyse the load capacity by considering different porous structure models such as the globular sphere model and capillary fissures model. Based on the calculated results, the preference can be suggested that which model gives much better performance for load capacity.

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