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Electric Charge and Electric Field Modeling Classical Theory, Computational Methods, and Multiscale Challenges

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Electric charge and the associated electric field are foundational to classical and modern physics and underpin electrostatics, electromagnetism and a wide range of engineering applications. This critical review synthesizes peer-reviewed literature published between 1990 and 2026 with an emphasis on classical and continuum electrostatic theory, computational and numerical modelling, experimental characterization, and emerging AI-assisted and physics-informed methods. The analysis shows that classical formulations remain valid at continuum scales, but in heterogeneous, interfacial, multilayer-dielectric and nanoscale systems they require hybrid continuum-particle, coupled-physics or surrogate-aided extensions. Numerical methods such as the finite-element method, boundary element method, finite difference method and particle-in-cell approaches are widely used, but their reported accuracy depends strongly on geometry, mesh resolution, dielectric contrast, and boundary specification and solver convergence criteria. Physics-informed neural network surrogates offer efficiency but remain limited in interpretability and out-of-distribution generalization. Experimental uncertainties arising from calibration drift, probe geometry, environmental noise and numerical post-processing propagate through the validation chain. Future progress will depend on the integration of rigorous physical theory, transparent uncertainty quantification, standardized benchmarks and physics-informed data-driven methods within coherent multiscale frameworks suited to electrostatic analysis in next generation technologies.

Keywords: Electric charge, Electric field, Electrostatics, Computational modelling, Finite-element method, Boundary-element method, Particle-in-cell method, Physics-informed neural networks, Machine learning, Multiscale modelling

1 Introduction

Electric charge and the electric field are foundational concepts of classical physics and form a central part of modern electromagnetic theory and engineering applications[1–5]. The field-based view replaced earlier action-at-a-distance interpretations with a spatially distributed description of charge–charge interactions[5–8], and these principles continue to inform the teaching and application of capacitors, semiconductors and nanoscale systems[8–10]. Recent experimental and computational developments have driven renewed interest in the study of electric fields in systems involving irregular geometry, heterogeneous materials and multilayer dielectric structures[9–13]. High-precision finite-element modelling has also been developed for field-emitter electrostatics[14]. Charged particulate systems with non-uniform field distributions have also been modelled[15]. In such cases numerical models including FEM and FDTD-type solvers are commonly used, but their outputs depend on geometry, boundary conditions, mesh resolution, dielectric contrast and solver convergence criteria. The same applies to BEM, PIC and physics-informed neural-network implementations[16–22]. Experimental validation of electrostatic models continues to be challenging at micro- and nanoscale regimes[23–31]. Related studies have examined time-varying electric-field generation and electrostatic adhesion in electrode-array geometries[24] and laser-induced plasma dynamics[26]. The growing overlap of computational electrostatics with first-principles materials modelling has been demonstrated in recent studies on halide perovskites[32], double perovskites[33], K_2CdSn and Rb_2CdSn alloys[34], $Cs_3Bi_2Br_9$ triple perovskite halide[35], perovskite hydrides[36], and actinide antimonides[37], all of which couple electrostatic field analysis with electronic-structure calculations. Figure 1 summarizes the conceptual relationship among electric charge, governing laws, analytical and computational approaches, and representative applications of electrostatics.

Despite these advances, several important challenges remain. First, there is still the absence of

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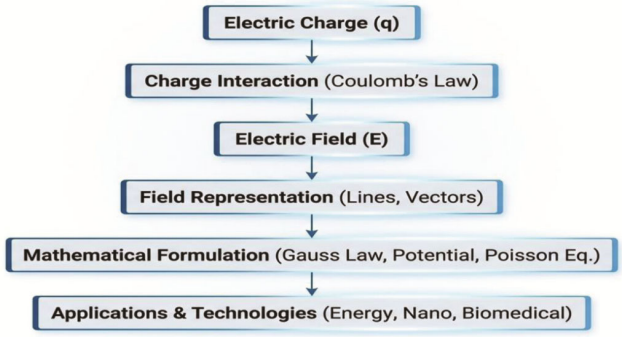


Fig. 1 — Conceptual relationship among electric charge, governing laws, analytical/computational approaches and applications of electrostatics; illustrated from the electrostatic-theory literature[1–5]

consolidated multiscale frameworks that couple continuum electrostatic theory with explicit atomistic and interface-dominated descriptions[2–4–22]. Second, the integration of established numerical techniques with emerging physics informed and machine-learning approaches remains limited[11–22]. Third, experimental validation of electrostatic models continues to be challenging at micro- and nanoscale regimes where continuum assumptions gradually lose validity[23–31]. Accordingly, this review critically examines the current understanding of electric charge and electric-field modelling. The discussion focuses on how electric charges and electric fields behave in non-uniform, dynamic and nanoscale systems where only part of the classical assumptions are satisfied a question that is central to reconciling idealised models with realistic charged-particulate and advanced-material systems[2–4–13–22] and on how electric fields can be studied through combined experimental and computational approaches, including contemporary numerical techniques and physics-informed machine-learning surrogates[9–22]. The review also considers how electric fields may be controlled and manipulated in future high-voltage, precipitation, nanomaterial, agricultural-electrostatic and biomedical-engineering applications[9–10–28–32]. By integrating theoretical, computational and experimental perspectives, it aims to provide a coherent assessment of the present status, limitations and future directions of electrostatic modelling.

2 Methodology

This review is based on a structured qualitative and interpretive analysis of the published literature and does not constitute a quantitative meta-analysis. The major scientific databases were searched to identify peer-reviewed journal articles and book chapters on electric charge and electric-field modelling, with emphasis on computational methods including FEM (finite-element method), BEM (boundary-element method), FDM (finite-difference method), PIC (particle-in-cell) techniques and machine-learning surrogates, as well as multiphysics, dielectric and semiconductor field analysis and nanoscale electrostatic measurements. The combinations of keywords used were: "electric field modelling", "electrostatics simulation", "finite-element electrostatics", "boundary-element method electrostatics", "particle-in-cell electrostatics", "physics-informed neural network electrostatics" and "electrostatic experimental validation". Studies were included when they (i) were published in peer-reviewed journals or scholarly books, (ii) appeared between 1990 and 2026, (iii) contained sufficient methodological transparency, and (iv) addressed theoretical, computational or experimental aspects of electrostatic modelling, multiphysics analysis or data-driven surrogate modelling. Conference abstracts lacking adequate methodological detail, non-peer-reviewed publications and purely pedagogical discussions without direct methodological relevance were excluded from the review. A total of 50 records were initially identified. Following duplicate removal, title and abstract screening, and full-text eligibility assessment, 37 studies were selected for qualitative synthesis. Mathematical expressions, computational results and numerical values discussed throughout this review represent benchmark-specific observations reported in the cited literature and should not be interpreted as universal electrostatic constants unless explicitly stated. The literature selection procedure adopted in this review is summarised in Fig. 2.

2.1 Essential Ideas of Electric Charge and Electric Field

Electric charge is a quantised and conserved property of matter, and the interaction between two point charges in vacuum is described by Coulomb's law;

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \quad \ldots (1)$$

Here ε_0 is the permittivity of free space. The electric field is the force per unit positive test charge,

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \ldots (2)$$

and is a vector field[1–5]. The principle of superposition states that the field of an arbitrary charge distribution equals the vector sum of the fields of the constituent

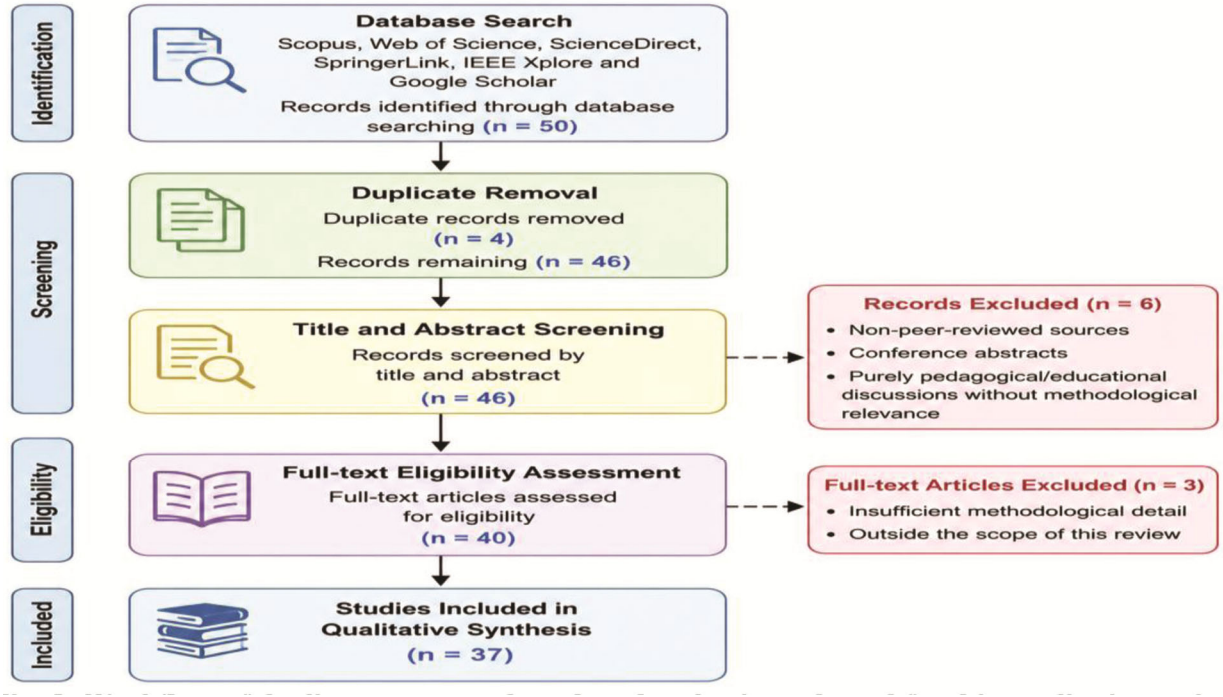


Fig. 2 — Workflow of the literature search and study selection adopted for this qualitative review

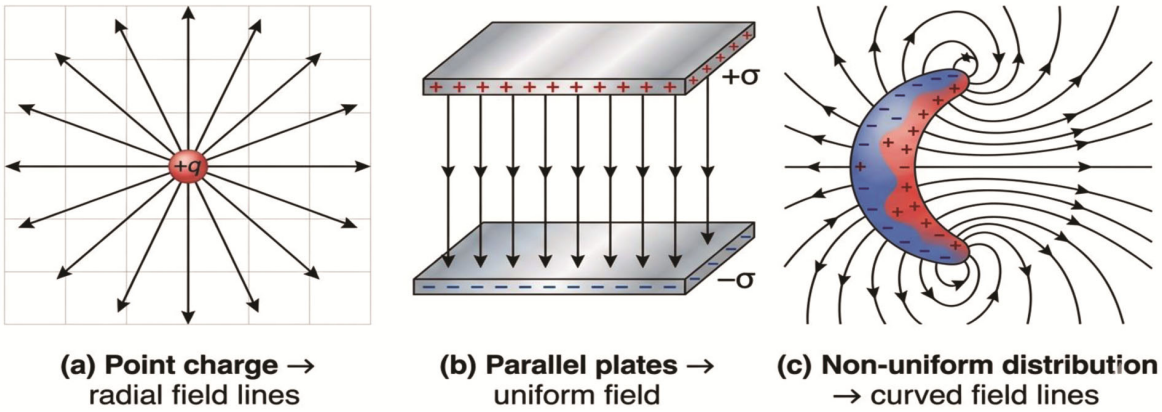


Fig. 3 — Schematic representation of electric-field distributions due to a point charge, uniform charge distribution and non-uniform charge system[1–5]

charges and applies to point, line, surface and volume charge densities[3–5]. Recent work has also explored fields of complex-shaped charge loops[27], demonstrating that classical superposition continues to provide the analytical baseline even for non-trivial charge geometries. Figure 3 illustrates characteristic field patterns of point, uniform and non-uniform charge distributions.

2.2 Field Equations and Mathematical Formulation

Mathematical modelling of electric charges and electric fields is central to classical electrostatic analysis[1–8]. The field is a vector quantity created by charges in space and is governed by the relevant physical principles and differential equations, with the resulting equations forming the starting point for analysis in systems where static fields and continuum simplifications are valid approximations[3–8].

2.2.1 Electric Field of Discrete and Continuous Charge Distributions

The electric field produced by a point charge at distance r is;

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\ \frac{q}{r^2}\ \hat{r} \qquad \ldots (3)$$

Table 1 — Fundamental laws governing electric charge and electric field

Law	Mathematical form	Key concept	Typical application
Coulomb's law	$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$	Force between charges	Particle interaction
Gauss's law	$\oint E \cdot dA = Q/\varepsilon_0$	Field–charge relation	Symmetric enclosures
Superposition	$E = \Sigma E_i$	Additivity of fields	Multi-charge systems
Potential theory	$\vec{E} = -\nabla V$	Scalar representation	Boundary-value problems

Source: Author's compilation based on classical and modern electromagnetism[1–8].
Note: Entries are canonical relations and conceptual summaries and are not benchmark performance values.

where $\hat{r}$ denotes the unit vector directed from the charge to the observation point. The superposition principle then allows the field of any distribution of charges to be computed by vector summation[3–5]. for a continuous volume charge density,

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')(r-r')}{|r-r'|^3}dV' \quad \ldots (4)$$

These integral expressions are also applied to line and surface charge distributions and remain relevant for analysing practical systems, including charged particulate materials and dielectric slabs[10–15–29].

2.2.2 Gauss's Law

Gauss's law relates the electric flux through a closed surface to the enclosed charge;

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \quad \ldots (5)$$

The corresponding differential form is;

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \ldots (6)$$

The differential form emphasizes the local relation between charge density and field and constitutes one of the four Maxwell equations[2–3]. The practical value of Gauss's law is most evident in problems with symmetry[5–6].

2.2.3 Electric Potential and Potential Energy

The electric potential at a point is the work required to bring a unit positive test charge from infinity to that point;

$$V(r) = -\int_{\infty}^{r} \vec{E} \cdot \vec{dl} \quad \ldots (7)$$

The field obtained through the negative gradient is;

$$\vec{E} = -\nabla V \quad \ldots (8)$$

These relations are widely used in energy-based analysis of capacitive and electrostatic storage systems and are particularly relevant to engineering boundary-value problems[3–5].

2.2.4 Poisson and Laplace Equations

In a region containing volume charge density;

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \quad \ldots (9)$$

In a charge-free region, the equation reduces to Laplace's equation[3–5];

$$\nabla^2 V = 0 \quad \ldots (10)$$

Coulomb, Gauss and Poisson descriptions collectively form a continuum field theory that is valid at macroscopic scales[2–4–5]. The continuum hypothesis weakens at micro- and nanoscale regimes, where discrete charge localisation, dielectric heterogeneity, interfacial polarisation and surface-dominated effects become significant[2–4–13–22]. In such cases interface-aware constitutive relations, effective-medium corrections or hybrid particle–continuum couplings are required to extend the classical equations across length scales[2–4–22]. Accordingly, the Poisson and Laplace equations are treated in this review as complementary representations of the same phenomenon rather than as a single universal law valid at all scales. Laplace's equation is obtained as the special case of Poisson's equation when the charge density is zero. The fundamental laws governing electric charge and electric fields are summarised in Table 1.

2.3 Experimental Techniques

Accurate measurement of electric charge and electric fields is essential for validating theoretical models and numerical simulations. Experimental characterisation has been a central pillar in the development and validation of electrostatic theory at all length scales. The classical experiments of Coulomb and Millikan set the quantitative foundation, while recent developments have introduced electro-optic, scanning-probe and time-varying-field techniques capable of capturing rapid, transient and nanoscale phenomena[23–31]. Theoretical analysis of time-varying electric fields generated by interdigital

Table 2 — Experimental methods used in electrostatic characterization

Technique	Measurement principle	Typical scale	Typical use
Torsion balance	Force measurement	Macro	Coulomb's law verification
Millikan oil-drop experiment	Charge quantisation	Micro	Elementary charge
Interdigital electrode array	Time-varying field generation	Macro/Meso	MEMS, adhesion[24]
Electro-optic and laser-plasma spectroscopy	Spectroscopic detection of field-induced plasma	Micro/Meso	Plasma dynamics[26]
Charged-aerosol field characterization	Sampling and field mapping	Field-scale	Atmospheric dust studies[28]
Liquid-flow electrification probes	Charge accumulation measurement	Lab/Industrial	Flow electrification[29]

Source: Author's compilation based on classical and modern experimental electrostatics literature[23–31].
Note: Methods differ in scale, uncertainty structure and measurement principle. Direct cross-study quantitative comparison should be made with caution because calibration protocols, probe geometry and signal-processing methods differ between laboratories.

Table 3 — Applications of electric fields in scientific and engineering domains

Domain	Representative technology	Functional role of the field
Energy storage	Capacitors and supercapacitors	Charge–voltage storage and rapid discharge
Industrial precipitation	Electrostatic precipitators[28]	Particle removal and emissions control
Agriculture	Electrostatic spray and biostimulation[27]	Charged-droplet deposition and pulsed-field treatment
MEMS	Cantilever and electrode actuators[24]	Field-driven actuation and sensing
Biomedicine	Electrophoresis, neural stimulation	Biological interaction and therapeutic application

Source: Author's synthesis based on applied electrostatics research[9–10–27–28].
Note: Conceptual summary; performance figures behind each application are problem-specific and not directly comparable.

electrode arrays has been combined with experimental measurement of electrostatic adhesion forces, with applications in MEMS and adhesion devices[24]. The radiated electric field of charged atmospheric aerosols generated from charged saltating dust aggregates has been characterised in atmospheric-dust environments[28]. Liquid-flow electrification has been extensively reviewed as a phenomenon relevant to industrial electrostatic safety[29]. Laser-induced plasma and bubble dynamics in external electric fields have been studied by spectroscopic techniques[26]. The increasing accuracy of modern electrostatic measurements derives from refined probe geometry, calibration procedures and signal-processing methods[23–31]. However, experimental studies remain limited by environmental noise, restricted spatial resolution and heterogeneous calibration protocols, all of which complicate direct cross-study comparison[28–31]. The principal experimental methods used for electrostatic characterisation are summarised in Table 2.

2.4 Applications of Electric Charge and Electric Field

Electrostatic concepts find broad application across energy storage, MEMS, electrostatic precipitation, agriculture and biomedical engineering[9–10–28]. In industrial electrostatic precipitation, recent progress has been reported in modelling particle-collection efficiency, corona discharge and back-corona instabilities[28]. In agricultural engineering, electrostatics research has progressed from macroscale spray engineering to microscale plant biostimulation, including electrostatic atomisation, charged-droplet deposition and pulsed electric-field treatment of plant tissues[27]. In nanotechnology, stress distribution and charge-density-induced stress in dielectric slabs have been analysed using combinations of FEM and analytical models[10]. Layered dielectric systems have been characterised under polarity-reversal conditions, in which interfacial charge accumulation drives failure and ageing[13]. Representative applications of electric fields across scientific and engineering domains are summarised in Table 3.

2.5 Computational Studies of Electric Charge and Electric Field Modelling

The increasing complexity of modern electrostatic systems has made computational modelling an indispensable tool for analysing electric-field distributions that cannot be solved analytically. Numerical methods provide efficient solutions for problems involving irregular geometries, heterogeneous dielectric materials, nonlinear boundary conditions and coupled multiphysics phenomena. Consequently, computational electrostatics has become an integral component of research in electrical

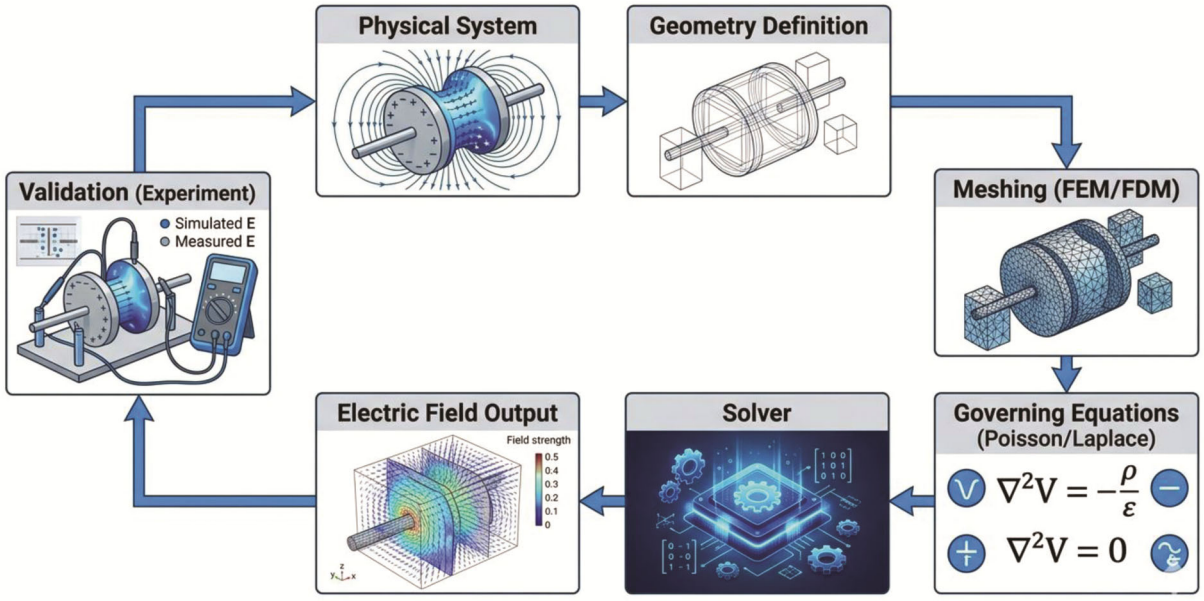


Fig. 4 — Workflow of computational modelling for electric-field analysis using numerical methods such as FEM, BEM and FDM[11–22]

engineering, materials science, plasma physics and nanotechnology[10-22].

2.5.1 Computational Workflow

A typical computational electrostatic analysis involves the definition of the physical geometry, assignment of material properties, specification of boundary conditions and discretisation of the computational domain. The governing electrostatic equations are subsequently solved using appropriate numerical techniques, followed by convergence assessment and validation against analytical solutions or experimental observations wherever possible. Figure 4 illustrates the general computational workflow adopted in electrostatic simulations.

2.5.2 Finite-Element Method

The finite-element method (FEM) is widely used to solve Poisson and Laplace equations in systems with non-trivial geometry, heterogeneous dielectrics and complex boundary conditions[14–17–22]. FEM is also used to determine electric-field strength and potential gradients in practical systems, including high-voltage insulation, capacitor design and microelectronic devices[14–15]; its popularity stems from flexible treatment of non-homogeneous materials and irregular geometries[11]. Numerical accuracy depends on the specified boundary conditions, mesh quality, element order and convergence tolerance, so FEM is best regarded as a powerful approximation technique rather than an exact solution[14–15–22].

2.5.3 Numerical Sensitivity and Boundary-Condition Dependence

The accuracy of FEM simulations depends critically on mesh refinement at electrode edges, dielectric interfaces and regions of high field gradient[14–15]. Reported FEM deviations of approximately 10–18 % and localised field-amplification factors of 1.2×–2.5× appear in specific benchmark configurations, but these figures are not universal electrostatic constants; they reflect the geometry, interface conditions and solver settings of the benchmark on which they are reported. Higher deviations are generally associated with strong dielectric discontinuities and complex boundary conditions, while lower deviations are reported when refined meshes and tight convergence criteria are used[14–15–22]. Particle-in-cell simulations, in which the electrostatic field is updated self-consistently with particle motion under space-charge effects, introduce additional sensitivity to particle-per-cell counts and temporal discretisation[21]. Reported numerical results should always be accompanied by a mesh-convergence or particle-resolution study, a transparent dielectric-property specification and a clear description of solver settings.

2.5.4 Boundary Element, Finite-Difference and Particle in Cell Methods

The boundary-element method treats integral equations on the boundary and is particularly advantageous for open-region problems such as exterior fields, floating conductors and high-aspect-ratio electrodes[17–22].The finite-difference method is

Table 4 — Conceptual comparison of major computational approaches for electrostatic analysis

Method	Representative benchmark	Typical error trend	Principal strength	Principal limitation
Closed-form Coulomb baseline	Symmetric point-charge benchmark	No discretisation error under ideal assumptions	Closed-form reference	Restricted to idealised geometry
FEM	Complex dielectric geometry with mixed boundaries[14–15]	Error decreases with mesh refinement; benchmark-specific	High spatial fidelity on irregular domains	Sensitive to mesh and boundary conditions
FDM/BEM	Open-boundary conductor benchmark or constrained BEM[17–20]	Boundary- or grid-discretisation error is problem-specific	Efficient for exterior-field or simple-geometry problems	Harder to apply to strongly heterogeneous or nonlinear media
PIC[21]	Single-emitter or array electrospray benchmark	Sensitive to particle-per-cell and time step	Self-consistent space-charge evolution	Cost scales with particle count
Neural predictor[11]	Coplanar device benchmark	Problem-specific MAE on held-out cases	Rapid surrogate prediction	Limited interpretability and out-of-distribution risk
Active-learning/deep surrogate[22]	Laplace-domain parameterised benchmark	Surrogate-acceleration gains over full FEM	Data-efficient design exploration	Data-driven fidelity
First-principles/electrostatic coupling[32–37]	Halide perovskite and antimonide alloy benchmarks	DFT-level fidelity; coupling is problem-specific	Atomic-scale electrostatics and optoelectronic properties	Computational expense of full DFT

Source: Author's synthesis based on computational and machine-learning literature[11–22–32–37].
Note: Typical error trends are conceptual and benchmark-specific. Reported numerical accuracy depends on the benchmark, mesh or training data, solver settings, boundary conditions and error metric. Cross-method comparison is meaningful only when benchmark and metric are clearly specified.

well adapted to problems with relatively simple geometries and is generally inexpensive to implement[20]. Physical implementations of these methods are used together with FEM in many practical problems, including open-boundary conductor benchmarks, structured-grid Laplace/ Poisson benchmarks and single-emitter or array electrospray PIC benchmarks[17–21]. Constrained BEM formulations have been proposed to reduce the deviation observed in standard BEM analyses of typical electrostatic-field problems[17]. Particle-in-cell methods have been used to extend single-emitter electrospray emission models to array scales, demonstrating that inter-particle forces introduce measurable plume-half-angle increment at high fractions of emitter current[21].

2.5.5 Physics Informed and Machine-Learning Surrogates

Several recent developments are noteworthy. (i) Active-learning frameworks have demonstrated that deep-neural surrogates can solve PDEs arising in electrostatic and electrostatic-optical design with significantly reduced labelled-data requirements[22]. (ii) Self-supervised physics-informed neural networks with parameterisable boundary conditions have been developed for parameterised Laplace problems and coplanar device-field modelling[11]. (iii) Coupled computational electrostatics and first-principles calculations have been performed on halide perovskite alloys[32–36], halide double perovskites[33–35], perovskite hydrides[36] and actinide antimonides[37], providing a template for connecting electrostatic modelling with atomic-scale material analysis. The accuracy of these surrogates depends strongly on training-data quality, they are often limited in interpretability and they tend to generalise poorly outside their training distribution[11–22]. A conceptual comparison of the major computational approaches used in electrostatic analysis is presented in Table 4.

2.5.6 Multiphysics and Coupled Simulations

In systems where electric fields generate thermal, mechanical or chemical responses, multiphysics frameworks are required[24–26–31]. Electrostatic actuators and semiconductor devices exhibit such coupling, and breakdown or failure analyses must account for electro-thermal and electromechanical interactions[13–24].

Figure 5 illustrates the typical multiphysics interactions of an electric field. In atmospheric dust studies, electrostatic interactions couple with aerosol transport, while in charged particulate systems electrostatic charging influences particle dynamics and deposition[28].

2.5.7 Emerging Computational Trends

Beyond accelerator-style surrogates, electrostatic concepts continue to find new applications in first-principles materials modelling. First-principles analyses of electric-field effects have been performed on halide

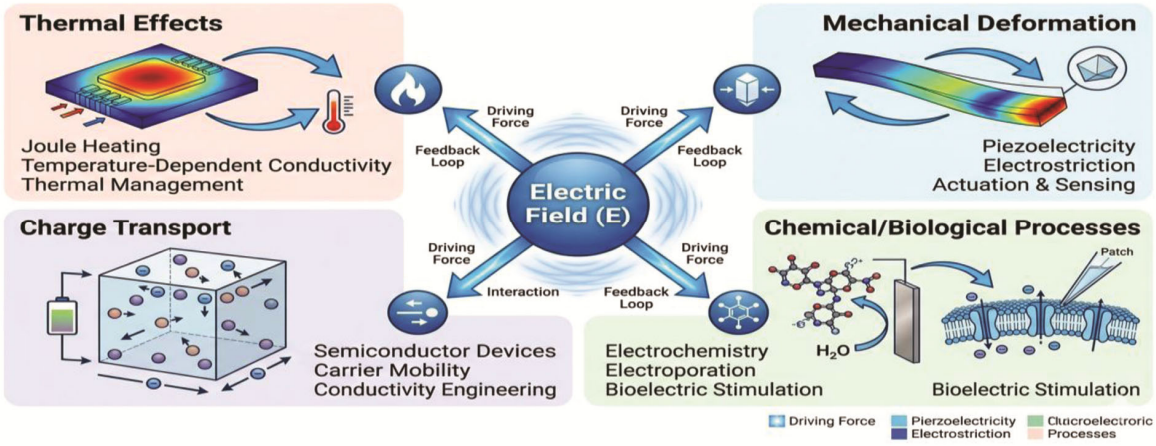


Fig. 5 — Multiphysics interactions of electric fields with thermal, mechanical and chemical processes[24–26–31]

double perovskites[33–35], perovskite hydrides[36], K_2CdSn and Rb_2CdSn alloys[34], $Cs_3Bi_2Br_9$ triple perovskite halide[35], spin-polarised K_2NaXI_6 double halide perovskites under electric-field perturbation[37], and actinide antimonides U_3TSb_5[37]. Charge-density-driven stress in dielectric slabs has been characterised using FEM in combination with analytical models[10], and interfacial charge accumulation under polarity reversal conditions has been modelled in multilayer dielectric systems[13]. These developments collectively suggest that electrostatic analysis is increasingly forming a structural component of multiscale electronic-structure and transport calculations, alongside its traditional role in device and field-emission analysis[14–22–32–37].

3 Discussion

The literature reviewed in this study demonstrates that classical electrostatic theory continues to provide the fundamental framework for analysing electric charge and electric-field behaviour across a broad range of scientific and engineering applications. Although Coulomb's law, Gauss's law and the Poisson-Laplace equations remain the theoretical foundation of electrostatics, modern technological applications increasingly require computational and experimental approaches capable of addressing complex geometries, heterogeneous materials and multiscale physical phenomena[1–37]. The review further indicates that substantial progress has been achieved through the integration of analytical theory with numerical simulation techniques such as the finite-element method (FEM), boundary-element method (BEM), particle-in-cell (PIC) methods and multiphysics computational frameworks. These approaches have significantly improved the capability to predict electric-field distributions in engineering systems that cannot be analysed using closed-form analytical solutions alone[10–22]. Nevertheless, several important challenges remain and continue to influence the reliability and applicability of electrostatic modelling.

3.1 Experimental Validation

One of the principal limitations identified throughout the literature is the difficulty of experimentally validating numerical electrostatic models, particularly in micro- and nanoscale systems. Although advanced techniques such as Kelvin probe force microscopy, electrostatic force microscopy and optical diagnostic methods have substantially improved spatial resolution, direct comparison between experimental measurements and numerical predictions remains challenging because of instrumental limitations, probe-induced perturbations and environmental noise[23–31]. Consequently, rigorous experimental validation remains essential for assessing the accuracy of computational electrostatic models and for establishing confidence in numerical predictions intended for engineering applications.

3.2 Measurement Uncertainty

Experimental characterisation of electric fields is intrinsically associated with measurement uncertainty from both systematic and random sources[28–31]. Systematic uncertainties may originate from calibration drift, probe geometry, instrument bias, the perturbative effect of the measuring device on the field and the limitations of calibration standards[29]. Random uncertainties arise from thermal fluctuations, electromagnetic interference, electronic noise and temporal signal variations. Additional uncertainty is introduced during numerical reconstruction, spatial

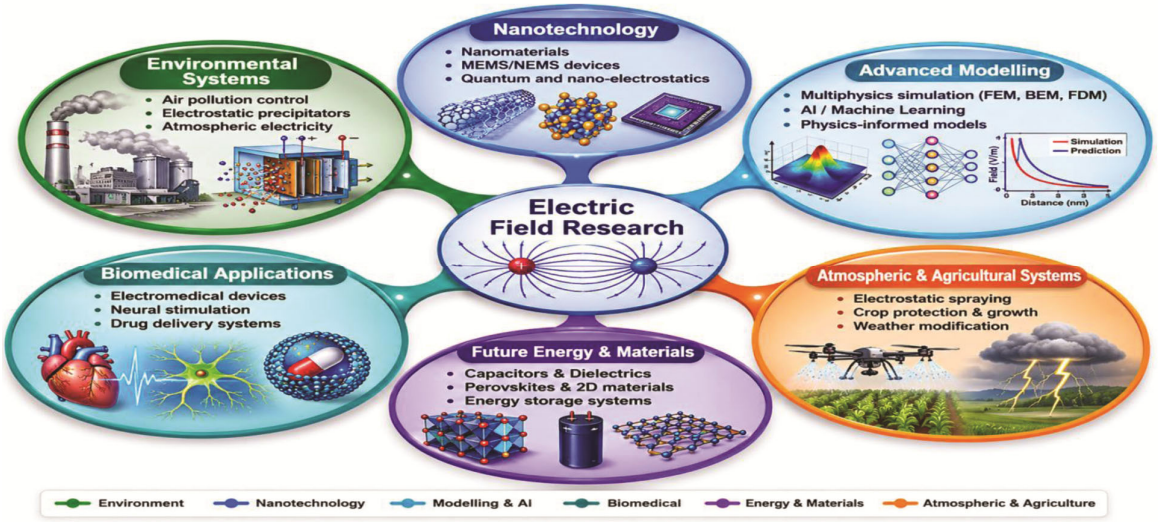


Fig. 6 — Emerging research directions in electric-charge and electric-field studies across interdisciplinary domains, including first-principles materials modelling[32–37], atmospheric and agricultural electrostatics[27–28], and AI-assisted modelling[11–22]

interpolation, digital filtering and post-processing of experimental data, and these uncertainties propagate through the measurement chain, influencing the comparison with computational predictions[29]. Direct comparison of results between studies should therefore be performed with caution because the underlying calibration protocols, probe designs and filtering methods differ between laboratories. Standardised uncertainty reporting, in which calibration uncertainty, spatial-resolution limits, repeatability and uncertainty propagation are explicitly quantified, is essential for reproducibility and meaningful cross-study comparison[28–31].

3.3 Classical Limitations of Electrostatics

The classical theory of electrostatics is a self-consistent and physically useful model for non-quantum systems indicated by modern continuum electrodynamics[2]. Its standard assumptions of fixed charge distribution, linear material response and homogeneity are not always valid in real systems[8]; time-varying magnetically active media such as plasmas cannot be modelled by the static equations alone, and discrete charge localisation, quantum effects and surface-dominated interactions require extension at micro- and nanometre scales[2–4–22]. Recent first-principles work on halide perovskite alloys[32] and $Cs_3Bi_2Br_9$ under electric fields[35] advances the atomic-scale treatment of polarisation and electric-field coupling, offering an alternative route to classical approaches when the continuum hypothesis breaks down.

3.4 Open challenges

Open challenges include (i) breakdown prediction in complex multilayer dielectrics under high fields[13]; (ii) scalable multi-scale electrostatic modelling without unacceptable loss of accuracy[18–22]; (iii) measurement of weak, transient and nanoscale fields[26–31]; (iv) standardised coupling of physics-based and AI-based methodologies in a manner that preserves interpretability[11–22]; and (v) integration of continuum, first-principles and machine-learning approaches for halide and oxide perovskites[32–37], perovskite hydrides[36], double perovskites[33–35] and actinide antimonides[37].

3.5 Future direction

Future progress in electrostatic modelling will depend on the integration of modern continuum electrodynamics[2], first-principles methods in finite electric fields[32–37] and machine-learning surrogates[11–22] with transparent experimental validation[23–31]. Hybrid physics-informed machine-learning models are particularly promising for accelerating electrostatic simulations while preserving physical consistency[11–22]. Future studies should further emphasise standardised benchmark problems, open uncertainty-quantification frameworks, multiscale coupling between continuum and atomistic models and high-resolution measurement techniques[23–31]. The integration of these elements is expected to enhance predictive capability in advanced dielectrics, MEMS, atmospheric electrostatic systems[9–10–28], and high-voltage systems[24–28]. The emerging interdisciplinary research directions that integrate continuum electrostatics, first-principles materials modelling, AI-assisted simulation, and atmospheric electrostatics and advanced engineering applications are summarised in Fig. 6.

4 Conclusion

Classical and modern continuum electrostatics continue to provide the fundamental framework for the analysis of electric charge and electric fields across scientific and engineering applications[1–8]. As this review has shown, realistic systems often involve heterogeneous materials, dielectric interfaces, multilayer insulation, multiphysics coupling and measurement uncertainty that extend beyond ideal continuum assumptions[2–4–13–22–28–31]. Computational techniques such as FEM, BEM, FDM, PIC and emerging physics-informed and machine-learning surrogates[9–22] should be regarded as complementary tools for analysing complex electrostatic phenomena rather than as universal solutions of the governing equations. This review demonstrates that classical formulations remain valid as a baseline but require hybrid extensions to capture interfacial, discrete, and quantum effects[2,4,13,22]. Regarding experimental and computational analysis, FEM, BEM, and PIC provide effective approaches for spatial modelling, while AI-based and physics-informed surrogates offer rapid evaluation, although their applicability depends on model formulation, training data, boundary conditions, and validation. Regarding control of electric fields in emerging technologies, the integration of reliable numerical modelling with physics-informed machine learning, standardised uncertainty reporting and advanced experimental probes[23–31] is the most promising direction for future progress. These developments are expected to advance first-principles materials modelling[32–37], atmospheric electrostatic systems[28], and high-voltage engineering[24–28]. Future consolidation of the field will therefore depend on the development of hybrid multiscale frameworks that integrate rigorous physical modelling, transparent uncertainty quantification, standardised experimental validation and physics-informed machine learning.

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