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Determination of the Critical Exponent for High Temperature Superconductors Using Paraconductivity Approach

M A Islam* & M R Islam

Department of Physics, University of Chittagong, Chattogram 4331, Bangladesh

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Theoretical ideas are developed to determine the critical exponent, understanding superconductivity for high-temperature superconductors. A detailed analysis is given based on the paraconductivity approach and compared with the experimental results. To develop the theoretical idea, the analysis is performed with the help of well-known paraconductivity expressions. To match the theoretical idea with experimental value, the data points were reproduced using the 'OriginPro' software. Numerical estimation has been given in favour of the samples $Tl_2Ba_2CaCu_2O_8$ (TBCCO), $Bi_{1.6}Pb_{0.4}Sr_2Ca_2Cu_3O_x$ (BSCCO), $YBa_2Cu_3O_{7-\delta}$ (YBCO) and $SmBa_2Cu_3O_{7-\delta}$ (SmBCO). The theoretical models largely agreed with the experimental results for the mentioned superconducting samples.

Keywords: Paraconductivity, Critical exponent, High-temperature superconductor

1 Introduction

The study of superconductivity due to the thermodynamic fluctuations has achieved a lot of interest in high-temperature superconductors. Fluctuation of these materials become very important because of their physical parameters like, high transition temperature, strong anisotropy and small coherence length. Therefore, it is a very necessary tool for understanding the intrinsic properties of superconductors. These study also provide the dimensionality of the order parameter fluctuation.

The critical exponent, denoted by λ is the most important exponent constant related to the dimensionality of the superconducting fluctuations. It is a pure (positive) number, also known as dimensional exponent (DE). The creation of Cooper pairs due to thermodynamic fluctuation above the superconducting transition with a finite lifetime will enhance electrical conductivity[1,2], so-called paraconductivity.

As the temperature approaches to the transition temperature T_c, the number of Cooper pairs increase while the normal electron density (NED) decreases. As a result, the resistivity decreases abruptly and the thermal fluctuations induce Cooper pairs give excess conductivity, denoted by $\Delta\sigma$. A major difference between normal conductivity σ and paraconductivity $\Delta\sigma$ is the nature of the charge carriers. The former carries current by a single electron and the latter carries current by a pair of electrons. The study of temperature dependent paraconductivity helps us to determine the critical exponent constant, determining the dimensionality of the superconducting systems.

*Corresponding author: E-mail: ashadphysics@gmail.com

2 Theoretical Model

The experimental observation of paraconductivity with temperature needs the exponent coefficient λ [which have the values 0.33, 0.5, 1.0, 1.5 etc.] as a fitting parameter. The critical exponent λ, whose values determine the dimensionality of the superconducting samples, can be found in the following way:

2.1 Method-I

By considering the classical argument of excesss conductivity (or fluctuation-enhanced conductivity) attributable to the direct acceleration of the superconducting pairs created by thermodyanmic fluctuations above T_c. If there are no fluctuations[3], the normal state dc conductivity is written as:

$$\sigma_n = \left(\frac{ne^2}{m}\right)\tau_{tr} \quad \ldots (1)$$

where, τ_{tr} is the transport relaxation time and n is the number density of the normal electrons.

By analogy, an ansatz was made that the superconducting fluctuations contribute to the additional conductivity $\Delta\sigma$, so-called fluctuation-enhanced conductivity or paraconductivity[4], can be written as:

$$\Delta\sigma = \left\langle |\Psi|^2 \right\rangle \left(\frac{e^{*2}}{m^*} \right) \left(\frac{\tau_K}{2} \right) \quad \ldots (2)$$

The equation obtained here for the conductivity has a Drude form, where $\langle|\Psi|^2\rangle$ is defined with the density of the superconducting electron pairs fluctuating in K mode and $\tau_K/2$ with the relaxation time of fluctuating Cooper pair.

The life time and the density of the superconducting fluctuation modes are given, while the sum over K can be converted to an appropriate integration depending on the dimensionality of the superconducting samples. In general, the sum can be replaced by integration for different dimensions as follows:

For bulk (3-D) materials

$$\sum_K \rightarrow \int \frac{d^3K}{(2\pi)^3} \quad \ldots (3)$$

For thin flim $[d << \xi]$ or (2-D) system

$$\rightarrow \frac{1}{d} \int \frac{d^2K}{(2\pi)^2} \quad \ldots (4)$$

For filament or (1-D) system

$$\rightarrow \frac{1}{s} \int \frac{dK}{2\pi} \quad \ldots (5)$$

Comparing with the standard integrals and performing the integration, the final results for paraconductivity are as follows:

$$\Delta\sigma^{3D}(T) = \frac{e^2}{32\hbar\xi(0)} \varepsilon^{-1/2}; \quad T > T_c \text{ [bulk sample, 3-D system]} \quad \ldots (6)$$

$$\Delta\sigma^{2D}(T) = \frac{e^2}{16\hbar d} \varepsilon^{-1}; \quad T > T_c \; [d << \xi, \; d\text{ - flim thickness, 2-D system]} \quad \ldots (7)$$

$$\Delta\sigma^{1D}(T) = \frac{\pi e^2 \xi(0)}{32\hbar s} \varepsilon^{-3/2}; \; T > T_c \; [s << \xi^2, \; s\text{ - cross-sectional area, 1-D system]} \quad \ldots (8)$$

where, $\varepsilon = (T - T_c)/T_c$ is called the reduced temperature and $\xi(T) = \xi(0)\varepsilon^{-1/2}$ has been used. Here, $\xi(0)$ is the zero temperature coherence length. The results obtained here for paraconductivity are well-known and essentially the same as first derived from microscopic theory by Aslamazov and Larkin (A-L)[5], so called Aslamazov-Larkin contribution.

To proceed, these expressions for paraconductivity are summarized in the following form:

$$\Delta\sigma \equiv A\varepsilon^{-\lambda} \quad \ldots (9)$$

where A is the temperature-independent amplitude which can be expressed as:

$$A = \frac{e^2}{32\hbar\xi(0)}; \quad \lambda = \frac{1}{2} \text{[3D system]}$$

$$= \frac{e^2}{16\hbar d}; \; \lambda = 1 \text{[2D system]} \quad \ldots (10)$$

$$= \frac{\pi e^2 \xi(0)}{32\hbar s}; \; \lambda = \frac{3}{2} \text{[1D system]}$$

Again, the temperature dependence of excess conductivity $\Delta\sigma(T)$, which is defined within the Ginzburg-Landau (G-L) mean field approximation[6] as:

$$\Delta\sigma(T) = \sigma_m(T) - \sigma_n(T) = \frac{1}{\rho_m(T)} - \frac{1}{\rho_n(T)} \quad \ldots (11)$$

where $\sigma_m(T)[=1/\rho_m(T)]$ represents the measured electrical conductivity and $\sigma_n(T)[=1/\rho_n(T)]$ is the linear normal-state conductivity. The normal-state resistivity of the sample $\rho_n(T)$ (is obtained from the measured resistivity $\rho_m(T)$ at $T \geq 2T_c$ by applying the least square method to the Anderson and Zou relation[7]:

$$\rho_n(T) = \rho_0 + \alpha T \quad \ldots (12)$$

where, α and ρ_0 are constants; $\alpha = d\rho/dT$ is the temperature resistivity coefficient which determines the slope of the linear dependence $\rho_n(T)$ and ρ_0 is the initial or residual resistivity (arising from the temperature-dependent scattering of electrons (charge carriers) by impurities, lattice defects (e.g. vacancies, dislocations), grain boundaries, and structural disorder within the material, rather than thermal vibrations) cut off by this line on the Y-axis[8-11] at $T = 0$. The linear term αT originates from electron-phonon scattering, which increases with temperature due to the growing phonon population.

Using Eq. (9) into Eq. (12), the result is obtained:

$$\frac{(\rho_n - \rho_m)}{\rho_m} = A\rho_n \varepsilon^{-\lambda} . \quad \ldots (13)$$

Now, taking derivative with respect to temperature on both sides of Eq. (13) and rearranging,

$$\frac{1}{\rho_m^2}\frac{d\rho_m}{dT} - \frac{1}{\rho_n^2}\frac{d\rho_n}{dT} = \frac{\lambda}{T_c^{MF}} A^{-1/\lambda}\left[\frac{\rho_n - \rho_m}{\rho_m \rho_n}\right]^{1+\frac{1}{\lambda}} \quad \ldots (14)$$

The critical exponent λ can be determined from the slope of a logarithmic plot of the left-hand side of Eq. (14) versus $\Delta\sigma = (\rho_n - \rho_m)/\rho_m \rho_n$.

2.2 Method-II

In this process, Eq. (9) is utilized through direct logarithmic operation. By taking the logarithm on both sides of Eq. (9), the resulting expression is:

$$Ln\Delta\sigma = Ln\left(A\varepsilon^{-\lambda}\right) = LnA - \lambda Ln\varepsilon$$
$$= -\lambda Ln\varepsilon + \text{constant} \quad \ldots (15)$$

where the searched value of critical exponent λ is equal to the negative value of the slope of the line fitted to the linear parts of this dependence. Here, the function $Ln\Delta\sigma$ strongly depends on the definition of the critical or transition temperature. The linear parts of Eq. (15) allows us to calculate the values of critical exponent λ.

2.3 Method-III

The third method is very useful to carry out the analysis of the experimental data using Eq. (14). It is convenient to apply the Kouvel-Fisher method[12], also known as the logarithmic derivative method. Thus, the expression takes the form:

$$\frac{d}{dT}(Ln\Delta\sigma) = \frac{d}{dT}\left[Ln\left(A\varepsilon^{-\lambda}\right)\right] = \frac{d}{dT}\left[LnA - \lambda Ln\varepsilon\right]$$
$$= -\lambda\frac{d}{dT}\left[Ln\varepsilon\right] = -\lambda\left(\frac{1}{\varepsilon}\right)\frac{d\varepsilon}{dT} = -\frac{\lambda T_c^{MF}}{T - T_c^{MF}}\frac{d}{dT}\left(\frac{T - T_c^{MF}}{T_c^{MF}}\right)$$
$$= -\frac{\lambda}{T - T_c^{MF}} \quad \ldots (16)$$

The inverse of the logarithmic derivative can be expressed as:

$$\chi_\sigma^{-1} = \left[-\frac{d}{dT}(Ln\Delta\sigma)\right]^{-1} = \frac{1}{\lambda}\left[T - T_c^{MF}\right] \quad \ldots (17)$$

This is a linear equation and it depends on the temperature of the superconducting sample only, which enables the direct determination of the mean-field transition temperature (T_c^{MF}) and critical exponent (λ) by identifying linear behaviors in curves of χ_σ^{-1} as a function of T. In the present work, the analysis focuses only on extracting the critical exponent from these linear behaviors. In this formula, the critical exponent is the inverse slope of the temperature dependence within linear regions and it is analogous to the Curie-Weiss susceptibility equation[13] in ferromagnetism, with the critical exponent λ playing the role of the Curie constant C.

By using Eq. (17), the numerical estimation as shown[14] in Fig. 1 for T_c^{MF} as well as λ is:

3 Results and Discussion

To determine the critical exponent λ, representative cases are considered based on the methods described in the previous section. With the

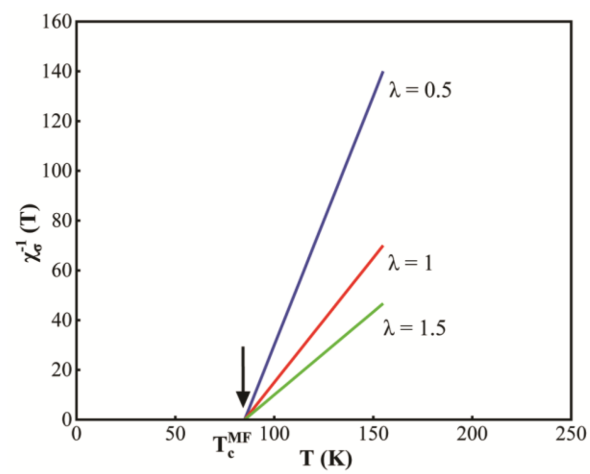

Fig. 1 — The plot of $\chi_\sigma^{-1}(T)$ versus T with different values of λ

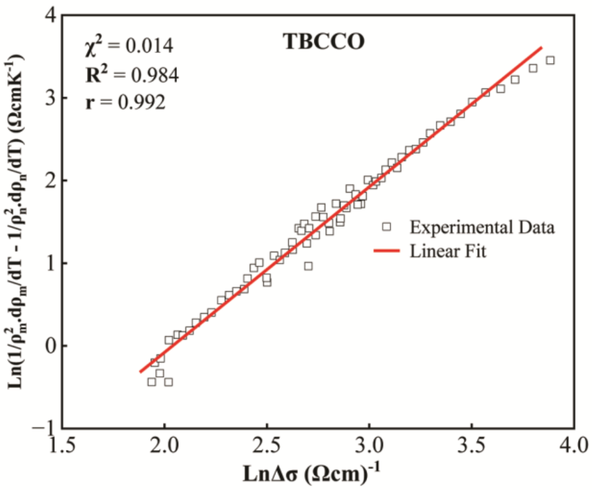

Fig. 2 — Logarithmic plot of the left-hand side of Eq. (14) versus excess conductivity $\Delta\sigma$ for the sample $Tl_2Ba_2CaCu_2O_8$ (TBCCO)[15]. A slope of 2 (2.002 ± 0.031) is observed over most of the range except for a very narrow region close to T_c, where the slope decreases [indicating an increase in the exponent in Eq. (13)]

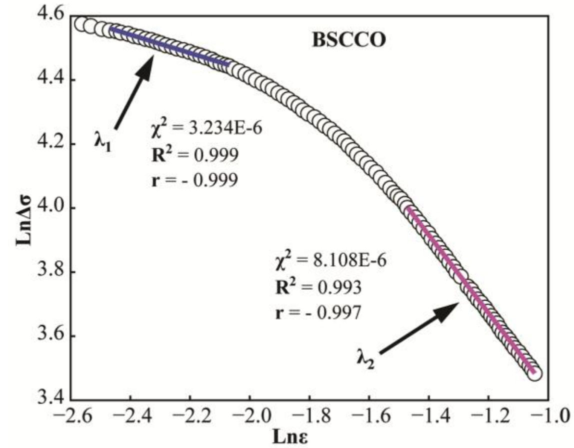

Fig. 3 — The $Ln\Delta\sigma$ versus $Ln\varepsilon$ dependence for $Bi_{1.6}Pb_{0.4}Sr_2Ca_2Cu_3O_x$ (BSCCO) tape. The straight lines fitted to linear regions give λ_1 and λ_2, critical exponents at the zero applied field. The curve is reproduced by 'OriginPro' software and the result was obtained $\lambda_1 = -0.286 \pm 0.004$ and $\lambda_2 = -1.217 \pm 0.002$ very close to the original values found experimentally[16]

help of method-I, the exponent λ can be determined from the slope of a logarithmic plot of the left-hand side of Eq. (14) versus $\Delta\sigma = (\rho_n - \rho_m)/\rho_m\rho_n$. The experimental data have been reproduced as shown[15] in Fig. 2 and fitted with the corresponding theoretical expressions. The analysis indicates that the Tl-Ba-Ca-Cu-O (TBCCO) system exhibits predominantly two-dimensional paraconductivity behavior near the superconducting transition temperature (T_c).

A further example[16] is taken (Fig. 3) for estimating λ described in method-II. In Fig. 3, the curve deviates from a single linear dependence and exhibits two well-defined linear regions, indicating that the fluctuation conductivity cannot be described by a single power law over the entire temperature range. According to the fluctuation conductivity theory, the linear behavior in the log-log plot confirms power-law scaling, where the slope of each linear segment corresponds to a critical exponent. The upper-left region (far from T_c) exhibits a relatively shallow slope, yielding the exponent λ_1 whereas the lower-right region (closer to T_c) shows a steeper slope corresponding to λ_2.

Finally, two examples[17,18] (Fig. 4) are presented using method-III discussed earlier. In Fig. 4, the experimental data points exhibit a non-linear behavior

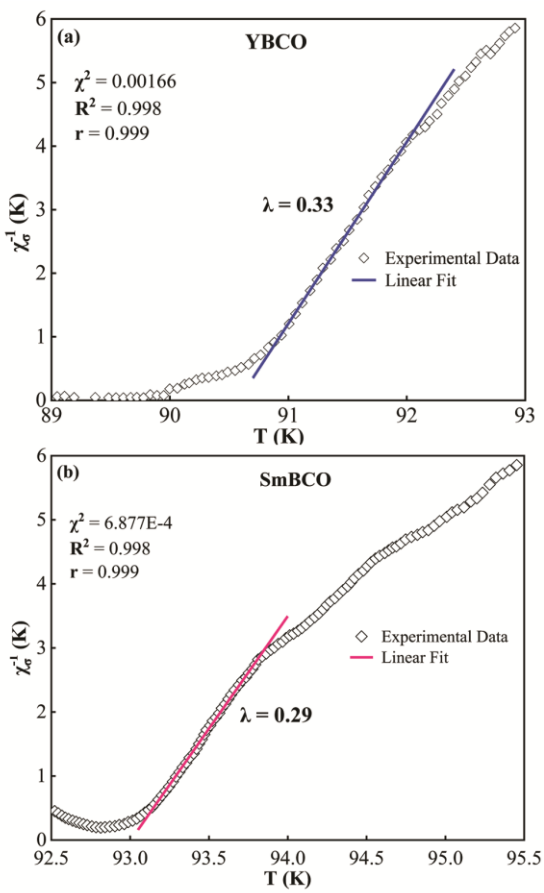

Fig. 4 — The plot of χ_σ^{-1} versus T (a) for sample $YBa_2Cu_3O_{7-\delta}$ (YBCO)[17] and (b) for sample $SmBa_2Cu_3O_{7-\delta}$ (SmBCO)[18]. The straight lines are the theoretical fit for estimating the value of λ

over the entire temperature range and a well-defined linear region is clearly observed in the vicinity of T_c. The slope of the fitted straight line provides the value of the critical exponent λ, regime consistent with three-dimensional (3D) behavior, as predicted by theoretical models which describes the nature of superconducting fluctuations. For the YBCO sample, the extracted value of $\lambda \approx 0.33$ ($\pm$0.024) indicates a fluctuation such as the Aslamazov-Larkin (A-L) theory. In comparison, the SmBCO sample yields a slightly lower value, $\lambda \approx 0.29$ ($\pm$0.022), indicating a weaker fluctuation regime.

4 Conclusion

By employing well-established fluctuation conductivity models and applying three independent methods, the critical exponent was extracted from the available experimental data using well-known paraconductivity expressions. The critical exponent constants obtained for the BSCCO tape (method-II) indicate distinct fluctuation regimes in these two regions (Fig. 3), whereas those obtained for the YBCO and SmBCO samples (Fig. 4) (method-III) suggests comparatively weaker fluctuations in SmBCO. The extracted critical exponent values provide meaningful insight into the dimensionality and nature of superconducting fluctuations in these materials. To estimate these constants, the published values of the related parameters for the mentioned samples were used. The goodness of fit is confirmed by a low chi-square (χ^2) value, a high coefficient of determination (R^2) and a Pearson's correlation coefficient (r) close to unity (as shown in the graph), indicating strong agreement between the theoretical models and experimental data. Future work can extend the present analysis to a broader class of superconducting materials (e.g., low-dimensional, anisotropic and strongly correlated systems), where fluctuation effects become significantly more dominant. The integration of numerical simulations and data-driven techniques may also enhance the precision and predictive capability of the analysis. Finally, we concluded that different theoretical ideas developed here are appropriate for determining the critical exponent constant of High Temperature Super Conductors (HTSCs).

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