Indian Journal of Pure & Applied Physics
Vol. 64, September 2026, pp. 955-960
DOI: 10.56042/ijpap.v64i9.22036

Odd-Even Staggering in Gamma Vibrational Bands of Deformed Nuclei Insights from Collective Models and Machine Learning Approaches

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Received: 26th July 2025; accepted: 3rd August 2026

The odd-even spin staggering of gamma-vibrational bands is examined for even-even Gd isotopes in the rare-earth region. The work uses experimental gamma-band energies, the empirical staggering index S(J), the energy ratio $R_{4/2}$, and the asymmetry parameter gamma0 to follow the evolution from gamma-soft behaviour toward more axially deformed rotational structures. The analysis is framed by collective-model ideas, especially the interacting boson model and the triaxial rotor picture, because both connect gamma-band level splitting with the softness or rigidity of the nuclear shape. A LightGBM regression procedure is further used only as a data-support tool for the missing S(7) and S(8) entries at N = 94. The imputed values are inserted with uncertainty estimates so that the observed trend of the Gd chain can be discussed without leaving artificial gaps in the systematics. The results show strong staggering near the transitional neutron numbers and a close relation between S(J), $R_{4/2}$ and asymmetry parameter γ_0. These observations support the view that gamma-band staggering is a sensitive indicator of triaxial effects, shape evolution and symmetry changes in deformed nuclei.

Keywords: Light GBM, Machine learning, Odd-even staggering, Gamma band, Asymmetry parameter, Gd isotopes, Collective models

1 Introduction

The low-lying spectra of even-even nuclei provide direct evidence for changes in nuclear shape, collectivity and symmetry. In the rare-earth region, Gd isotopes are especially useful because they lie between nearly vibrational and well-deformed rotational structures. Their ground-state and gamma-vibrational bands show systematic changes with neutron number, and these changes can be followed through the energy ratio $R_{4/2} = E(4_1^+) / E(2_1^+)$ and through odd-even spin staggering in the gamma band[1-5].

Odd-even spin staggering is the relative displacement of odd-spin gamma-band levels with respect to neighbouring even-spin levels. In gamma-soft nuclei, the grouping of levels is different from that expected for a rigid triaxial rotor; therefore, the phase and amplitude of the staggering pattern are widely used as signatures of gamma softness, triaxial rigidity and shape transitional behaviour. The staggering index S(J) is convenient because it uses neighbouring gamma-band energies and is normalized by the first 2^+ ground-state energy. In this form, it highlights the local irregularity in the gamma band without being dominated by the absolute energy scale[1,4,5].

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The physical interpretation of gamma-band staggering is closely related to the collective description of the nucleus. In the geometric model, the quadrupole shape is usually described by beta and gamma variables, where gamma measures the departure from axial symmetry[6]. The Davydov-Filippov model describes a rigid triaxial rotor, whereas the Wilets-Jean model represents a gamma-soft potential[7,8]. The interacting boson model (IBM-1) provides an algebraic framework in which the U(5), O(6) and SU(3) limits are associated with vibrational, gamma-soft and axially deformed rotational structures, respectively[9-14]. These models therefore offer a useful language for discussing the Gd isotope chain.

Earlier studies have shown that the staggering pattern in gamma-vibrational and quasi-gamma bands can distinguish different collective structures in even-even nuclei[1-5]. The asymmetric rotor model and related approaches have also been used to estimate the asymmetry parameter γ_0 and to connect it with energy ratios, transition strengths and shape evolution[15-23]. In the present work, the same physical direction is retained: the Gd isotopes are examined through S(J), $R_{4/2}$ and γ_0 in order to identify the structural behaviour of the gamma band.

A practical difficulty in such systematics is the absence of some experimental gamma-band energies

Table 1 — Empirical and imputed values of odd-even spin staggering in gamma bands of Gd isotopes, together with $R_{4/2}$ and γ_0. The two entries at N = 94 for S(7) and S(8) are LightGBM-supported imputed values with uncertainty bounds

N	S(4)	S(5)	S(6)	S(7)	S(8)	R4/2	Gamma 0 (degree)
88	-0.60	0.60	-0.51	0.51	-0.40	2.19	21.48
90	0.03	0.30	0.04	0.24	0.03	3.01	13.86
92	0.11	0.49	-0.16	0.77	-0.50	3.23	11.05
94	0.18	0.33	0.24	0.86 +/- 0.018	0.94 +/- 0.021	3.28	10.32
96	0.28	0.31	0.26	0.32	0.19	3.30	10.98

or derived staggering entries. To reduce this gap, the present work uses a LightGBM machine-learning model as an imputation tool for the missing entries in Table 1. The role of machine learning is limited to completing the missing S(7) and S(8) values at N = 94 by using nearby nuclear-structure observables as input features. The physics interpretation remains based on collective-model trends and experimental systematics, while LightGBM supports the continuity of the data table[24-26].

The novelty of the present manuscript, relative to the earlier work[24], is the combined discussion of Gd gamma-band staggering with $R_{4/2}$, γ_0 and LightGBM-supported imputation of missing values. The earlier paper introduced the staggering formulation for understanding shape transitions, whereas the present version focuses on the Gd chain, includes a clearer data-treatment procedure, and discusses the reviewer-requested links with IBM-1 and triaxial-rotor descriptions[24].

2 Formalism

2.1 Asymmetric Parameter

The asymmetry parameter gamma0 measures the deviation of the nuclear shape from axial symmetry. In the asymmetric-rotor description, γ_0 may be estimated from the ratio $R_\gamma = E(2_\gamma^+)/E(2_1^+)$. The expression used in Eq. (1)[22,27-29] is;

$$\gamma_0 = 0.33 \sin^{-1}[1.125\{1-(R\gamma -1/R\gamma +1)^2\}]^{1/2} \qquad \text{... (1)}$$

Here $E(2_\gamma^+)$ and $E(2_1^+)$ are expressed in keV and γ_0 is given in degrees. The limiting value γ_0. = 0 degree corresponds to an axially symmetric prolate shape, while γ_0. approaching 30 degrees represents the triaxial limit in the rigid-rotor picture[7,27].

2.2 Odd-Even Staggering

The odd-even staggering index for the gamma band is calculated as[1];

$$S(J) = \{E(J^+_\gamma) - 2E[(J-1)^+_\gamma] + E[(J-2)^+_\gamma]\} / E(2^+_1) \qquad \text{... (2)}$$

All energies in Eq. (2) are in keV; therefore, S(J) is dimensionless. Positive or negative values of S(J) indicate the relative displacement of the odd- and even-spin members of the gamma band. A small value suggests weak staggering, whereas a large oscillatory value reflects stronger gamma-band splitting and possible triaxial effects[1,4,5].

2.3 Energy Ratio $R_{4/2}$

The ratio $R_{4/2}$ is used as the standard indicator of quadrupole collectivity;

$$R_{4/2} = E(4_1^+) / E(2_1^+) \qquad \text{... (3)}$$

$R_{4/2}$ is dimensionless. Values near 2.0 are usually associated with vibrational nuclei, values near 2.5 with gamma-soft structures, and values approaching 3.33 with well-deformed axial rotors[6,9-12].

3 Data Set and LightGBM Treatment

3.1 Experimental Data

The empirical data set contains the Gd isotopes with neutron numbers N = 88, 90, 92, 94 and 96. Experimental level energies are taken from evaluated nuclear data tables and the NNDC/ENSDF database, and the values of S(J), $R_{4/2}$ and γ_0 are derived from Eqs. (1)-(3)[30].

3.2 LightGBM Imputation

The LightGBM model is not used here to replace nuclear theory. It is used only to estimate the missing S(7) and S(8) values at N = 94. The input features are neutron number N, spin J, $R_{4/2}$, γ_0, neighbouring staggering values S(J-1) and S(J+1), and the local gamma-band energy-spacing pattern. The target variable is the missing S(J) value. The model is trained as a regression model with L2 loss, 500 estimators, learning rate 0.05, 31 leaves, feature fraction 0.8, bagging fraction 0.8 and bagging frequency 5. Uncertainty is estimated from the spread of repeated LightGBM runs and cross-validation predictions[31-32].

The model gives S(7) = 0.86 $\pm$ 0.018 and S(8) = 0.94 $\pm$ 0.021 for N = 94. These values are used only to complete the trend shown in Table 1 and in Figs. 1-3. The imputed points are marked with uncertainty bars in the figures.

4 Collective-Model Parameters

4.1 IBM- 1 Description

The collective-model background is stated explicitly. In IBM-1, the low-lying collective spectrum may be described by a Hamiltonian containing the d-boson number operator and quadrupole-quadrupole interaction. Within the present framework, the IBM Hamiltonian is expressed as;

$$H_{IBM} = \epsilon_d \hat{n}_d - \kappa \hat{Q}^{\chi}\cdot\hat{Q}^{\chi} + \kappa' \hat{L}\cdot\hat{L} \quad \ldots (4)$$

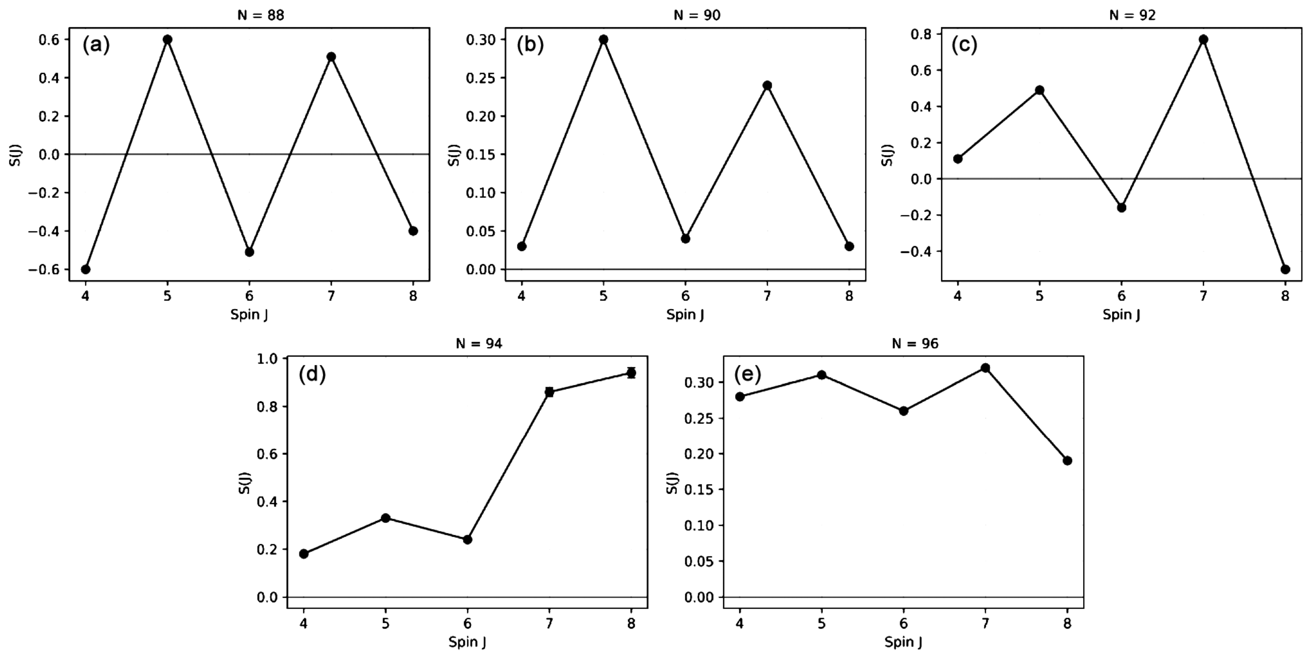

Fig. 1 — (a)-(e) Variation of empirical S(J) with spin J for Gd isotopes N = 88, 90, 92, 94 and 96. The N = 94 panel includes LightGBM-imputed S(7) and S(8) values with uncertainty bars

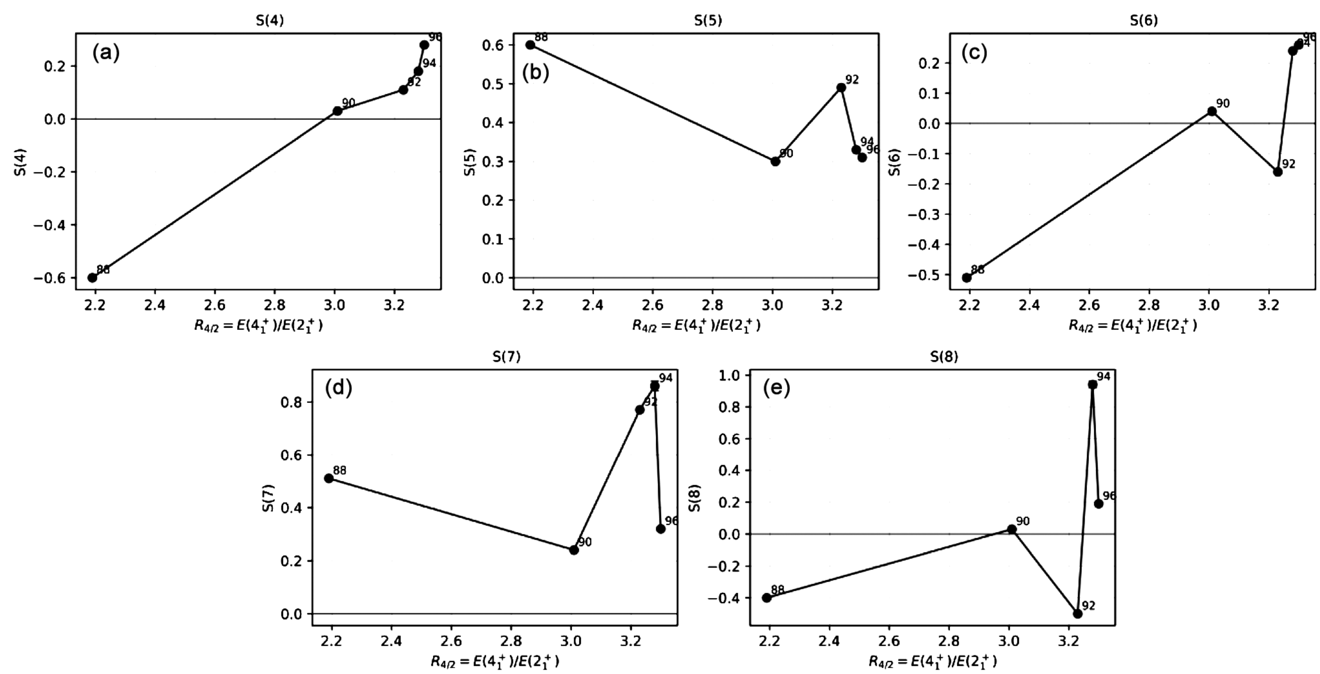

Fig. 2 — (a)-(e) Variation of S(4), S(5), S(6), S(7) and S(8) with the energy ratio $R_{4/2} = E(4_1^+)/E(2_1^+)$. Neutron numbers are marked near the data points

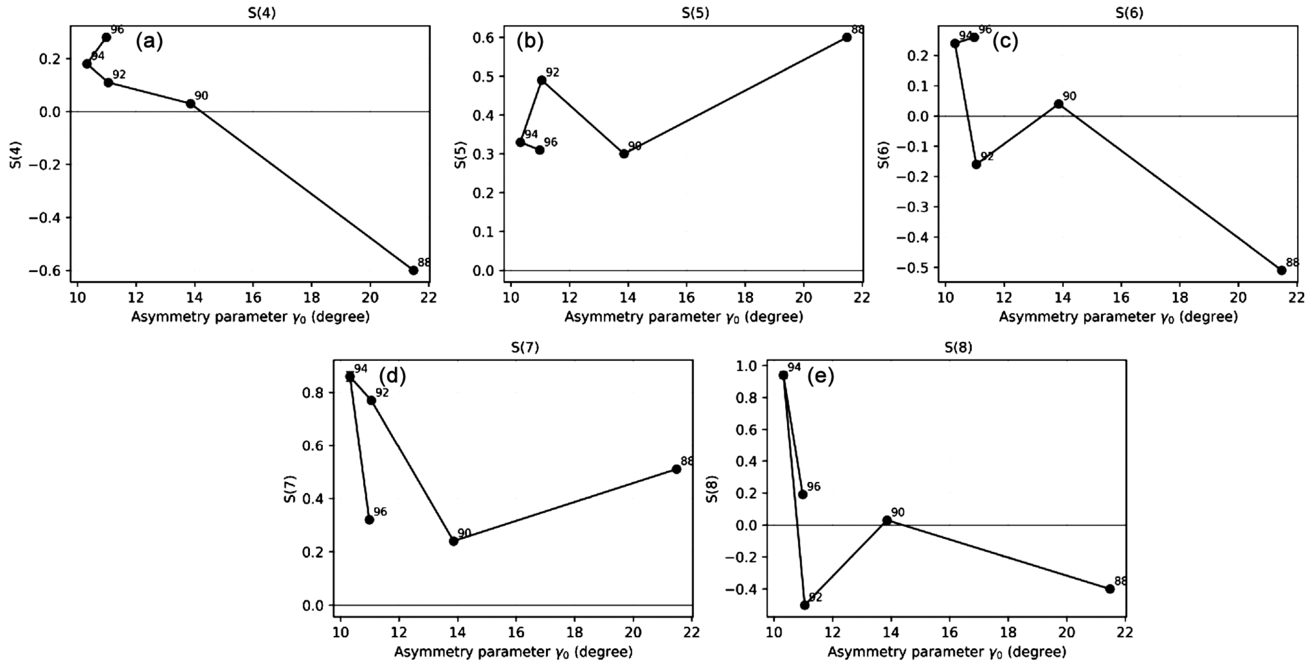

Fig. 3 — (a)-(e) Variation of S(4), S(5), S(6), S(7) and S(8) with the asymmetry parameter γ_0. The plots show how staggering changes as the Gd chain moves from larger γ_0 toward smaller γ_0 values

Table 2 — IBM-1 and triaxial-rotor interpretation of the Gd isotope systematics. The table keeps the original paper meaning: the parameters are used to support the experimental trend rather than to claim a full least-squares Hamiltonian fit[22,30]

N	R4/2 exp.	γ_0 (degree)	IBM-1 character	Triaxial-rotor indication	Main staggering feature
88	2.19	21.48	transitional/vibrational side	large gamma0; stronger triaxiality	large alternating S(4)-S(8)
90	3.01	13.86	near onset of deformation	reduced gamma0	moderate positive staggering
92	3.23	11.05	deformed SU(3)-like tendency	weak-to-moderate triaxiality	large S(7), negative S(8)
94	3.28	10.32	well-deformed rotor tendency	small gamma0	imputed large S(7), S(8)
96	3.30	10.98	rotational limit approached	small gamma0	weak positive staggering

For the Gd chain, the interpretation follows the standard evolution from a transitional U(5)-like region toward an SU(3)-like deformed rotor as N increases. The parameters are therefore used qualitatively: increasing quadrupole collectivity and decreasing γ_0 are associated with the observed rise of $R_{4/2}$ from 2.19 at N = 88 to about 3.30 at N = 96.

4.2 Triaxial Rotor Description

For comparison with the triaxial rotor picture, the rotor Hamiltonian is written as;

$$H_{rot} = A_1I_1^2 + A_2I_2^2 + A_3I_3^2 \quad \text{... (5)}$$

where A_k (K=1,2,3) and I_k are the principal moments of inertia[7,27]. The γ_0 values listed in Table 1 serve as empirical indicators of the triaxial character. Larger γ_0 values, such as γ_0 = 21.48 degree at N = 88, indicate stronger departure from axial symmetry, while values close to 10-11 degree for N = 92-96 indicate a more deformed but not perfectly axial structure. The corresponding IBM-1 and triaxial-rotor interpretations of the Gd isotopic systematics are summarized in Table 2.

5 Results and Discussion

5.1 Staggering Behaviour

The Gd isotopes considered here cover the structural region from N = 88 to N = 96. The value $R_{4/2}$ = 2.19 at N = 88 indicates a transitional or less-deformed structure, whereas $R_{4/2}$ rises to 3.01 at N = 90 and then approaches the rotational value for N = 92-96. This confirms the expected development of collectivity in the rare-earth region[33-35].

Figure 1 shows S(J) as a function of spin for each Gd isotope. At N = 88, the staggering pattern is strongly alternating, with negative S(4), positive S(5), negative S(6) and positive S(7). This behaviour reflects the irregular spacing of gamma-band levels in the transitional region. At N = 90, the values become smaller and mostly positive, which suggests a smoother gamma-band sequence. At N = 92, the large positive S(7) and negative S(8) again point to a pronounced staggering effect. For N = 94, the missing S(7) and S(8) values are completed through LightGBM imputation, and the resulting large positive values show that the local trend remains significant. At N = 96, the values become moderate, which is consistent with the approach to a more stable deformed structure.

5.2 Correlation with $R_{4/2}$

Figure 2 relates the staggering values to $R_{4/2}$. Since $R_{4/2}$ increases from 2.19 to about 3.30 along the chain, the plots show how the gamma-band staggering changes as the nucleus moves from a transitional region toward a rotational region. The correlation is not purely linear because S(J) is sensitive to local gamma-band splitting, but the change of amplitude around $R_{4/2}$ = 3.0-3.3 supports the interpretation of a shape transition.

5.3 Relation with Asymmetry Parameter

Figure 3 shows the relation between S(J) and γ_0. The decrease of γ_0 from 21.48 degree at N = 88 to about 10-11 degree for the heavier Gd isotopes indicates a gradual reduction of triaxial softness as deformation develops. However, the non-zero staggering values show that the gamma band does not behave as a perfectly axial rotor. Thus, the Gd isotopes retain gamma-degree sensitivity even in the more deformed region.

The combined behaviour of S(J), $R_{4/2}$ and γ_0 supports the same physical picture developed in the original manuscript: gamma-band staggering works as a sensitive marker of shape evolution. It identifies the competition between gamma-soft and gamma-rigid tendencies and links the level-spacing irregularity with collective deformation[1,5,7,8,24].

The LightGBM component is useful only for completing the missing N = 94 entries. It does not change the physical model; rather, it helps avoid an artificial break in the plotted systematics. The imputed values should therefore be read as data-assisted estimates, not as replacement for future experimental measurements[32].

5 Conclusion

The present study examines odd-even spin staggering in the gamma-vibrational bands of Gd isotopes by using the staggering index S(J), the collectivity ratio $R_{4/2}$ and the asymmetry parameter γ_0. The main physical result is that the Gd chain shows clear structural evolution from a more transitional region at N = 88 toward a well-deformed rotational region for N = 92-96.

The observed and imputed staggering values show that gamma-band level spacing remains sensitive to triaxial and gamma-soft effects. The decrease of γ_0 with increasing neutron number and the rise of $R_{4/2}$ support the interpretation of increasing deformation. At the same time, non-zero S(J) values indicate that the gamma degree of freedom still affects the level sequence.

The LightGBM method is used as a limited imputation tool for the missing S(7) and S(8) values at N = 94. This data-support step helps complete the isotopic trend but does not replace experimental measurements or the collective-model interpretation. The combined use of S(J), $R_{4/2}$, γ_0 and LightGBM therefore provides a compact way to discuss gamma-band staggering and shape evolution in deformed Gd nuclei.

Future experimental work should focus on confirming the gamma-band levels that control the S(7) and S(8) values at N = 94. More complete lifetime and B(E2) data would also help separate gamma softness from rigid triaxiality. On the computational side, the LightGBM method can be extended to a larger rare-earth data set, but the model should always be validated against newly measured levels before using the imputed values for strong physical conclusions.

References

1 McCutchan E A, Bonatsos D, Zamfir N V & Casten R F, *Phys Rev C*, 76 (2007) 024306.
2 Gupta J B & Kavathekar A K, *Pramana*, 61 (2003) 167.
3 Singh M, Bihari C, Singh Y, Gupta D, Varshney A K, Gupta K K & Gupta D K, *Can J Phys*, 85 (2007) 899.
4 Gupta J B, Sharma S & Katoch V, *Pramana*, 81(2013) 75.
5 Zamfir N V & Casten R F, *Phys Lett B*, 260 (1991) 265.
6 Bohr A & Mottelson B R, *Nucl Struct*, 1 (1998) 471.
7 Davydov A S & Filippov G F, *Nucl Phys*, 8 (1958) 237.
8 Wilets L & Jean M, *Phys Rev*, 102 (1956) 788.
9 Arima A & Iachello F, *Ann Phys*, 123 (1979) 468.
10 Arima A & Iachello F, *Phys Rev Lett*, 35 (1975) 1069.
11 Arima A & Iachello F, *Ann Phys*, 281 (2000) 2.
12 Iachello F & Arima A, (2006) The Interacting Boson Model, Cambridge University Press, 1998, pp. 264
13 Gneuss G & Greiner W, *Nucl Phys A*, 171 (1971) 449.
14 Gupta J B, *Pramana*, 89 (2017) 1.
15 Davidson J P, *Rev Modern Phys*, 37 (1965) 105.
16 Davydov A S & Rostovsky V S, *Nucl Phys*, 60 (1964) 529.
17 Gupta J B & Sharma S, *Phys Scrip*, 39 (1989) 50.
18 Mittal H M, Sharma S & Gupta J B, *Phys Script*, 43 (1991) 558.

19 Yan J, Vogel O, von Brentano P & Gelberg A, *Phys Rev C*, 48 (1993) 1046.
20 Esser L, Neuneyer U, Casten R F & von Brentano P, *Phys Rev C*, 55 (1997) 206.
21 Kumari P & Mittal H M, *Open Phys*, 13 (2015) 305.
22 Bindra A & Mittal H M, *Nucl Phys A*, 975 (2018) 48.
23 Bindra A & Mittal H M, *Chin Phys C*, 41 (2017) 054102.
24 Sharma P & Bindra A, *Indian J Pure & Appl Phys*, 62 (2024) 979.
25 Sharma P, Bindra A & Ahmad M, *Proceed DAE Sympos Nucl Phys*, 68 (2024) 231.
26 Ahmad M, Bindra A, Sharma P & Dogra V, *Proceed DAE Sympos Nucl Phys*, 68 (2024) 215.
27 Davydov A S & Rostovsky V S, *Nucl Phys*, 12 (1959) 58.
28 Varshni Y P & Bose S, *Nucl Phys A*, 144 (1970) 645.
29 Varshney V P, Gupta K K, Chaubey A K & Gupta D K *J Phys G*, 8 (1982) 727.
30 National Nuclear Data Center, Evaluated Nuclear Structure Data File (ENSDF), Brookhaven National Laboratory, https://www.nndc.bnl.gov/ensdf/.
31 Tonev D, Dewald A, Klug T, Petkov P, Jolie J, Fitzler A, *et al*., *Phys Rev C*, 69 (2004) 034334.
32 Lv B F, Li Z L, Wang Y J & Petrache C M, *Phys Lett B*, 857 (2024) 139013.
33 Casten R F, Brenner D S & Haustein P E, *Phys Rev Lett*, 58 (1987) 658.
34 Casten R F & Zamfir N V *Phys Rev Lett*, 87 (2001) 052503.
35 Kruecken R, Albanna B, Bialik C, Casten R F, Cooper J R, Dewald A, *et al*., *Phys Rev Lett*, 88 (2002) 232501.