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Lower Hybrid Wave Instability in A Magnetized Plasma Containing Two Ion Species

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In the current study, we have developed a theoretical model of a plasma cylinder having radius ~ 2 cm and considering electrons, light potassium (K^{+}) positive and heavy (Cs^{+}) positive ion species inside it. Through one end of the cylinder, an ion beam with energy of 10 eV and a radius of approximately 2cm is injected parallel to the static magnetic field. Fluid theory is applied to perform the numerical calculations of the frequency and the growth rate for both the positive unstable ion modes. The comparative analysis is performed between growth rate and the concentration of positive ions, and we found that it scales as the cube root of the ion beam density and rises as the relative concentration of positive ions increases. However, in comparison to the heavy positive ion mode, our growth rate increases more quickly for the light positive ion mode. Moreover, the unstable wave frequency and the corresponding wave number for both light and heavy positive ion modes are obtained by the point of intersection method. The results show that the unstable wave frequency increases with one-half power of ion beam energy for both the cases but the increase in unstable frequency is much greater for the light positive ion mode as compared to the heavy positive ion mode. As we know that the earth's ionosphere contains multi positive ion species and the concentration of these species alters the properties of the plasma present in ionosphere, therefore, the observations presented in this study may be helpful in understanding the factors affecting plasma.

Keyword: Lower hybrid wave instability; Unstable frequency; Two ion component; Growth rate

1 Introduction

The lower hybrid wave (LHW) instability driven with an ion[1] and electron beam[2,3] plays a vital role in laboratory as well as space plasmas. Waves in heliospheric plasmas are involved in many universal heliophysical phenomenon, for example, plasma heating, particle acceleration, and emission processes which include the process of energy releases. Lower hybrid (LH) waves are of particular interest because they involve both electrons and ions; these waves can transfer energy between parallel motions of electrons and perpendicular motions of ions. The lower hybrid wave mode driven by an ion beam in a radio frequency plasma is reported by Chang[1]. He reported maximum growth rate of the instability when the wave propagating along the magnetic field lines has the phase velocity comparable to the electron thermal velocity. He observed the electron heating phenomena also because of the instability. The phenomena of excitation of quasi-static LHW Eigen modes with the help of an electron beam having small density in finite geometry plasma is studied by Shoucri and Gagne[2] and reported that instabilities are present in the low frequency surface waves. Also, the finite geometry of the plasma column has an important effect on the linear excitation of these waves which is qualitatively and quantitatively different from the infinite geometry effects. The phenomena of excitation of LHW produced with the help of an electron beam is observed analytically by Papadopoulos and Palmadesso[3] and the authors observed and discussed the significance of such waves in tokamak plasma with runaway electrons in detail. Seiler *et al.*[6] have experimentally reported a spiralling ion beam driven lower hybrid wave instability in a linear Princeton Q-1 device and they found that the instability will occurs when frequency is above the cyclotron harmonics frequency. D' Angelo and Merlino[7] explained the EIC mode in a plasma contains electrons, positive and negative ions and the basics of lower hybrid wave instability in the existence of negative ions is reported by D' Angelo *et al.*[8]. The phenomena of excitation of LHW with ion streaming and in the existence of dust grain fluctuations is observed by

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Islam *et al.*[13] and they observed that the electrostatic lower-hybrid mode excited by the unmagnetized ions damp swiftly as compared to that due to the dust charge fluctuations. Prakash *etal.*[14] explained the electron beam driven lower hybrid wave instability in dusty plasma and they observed that with increasing the relative density of negatively charged dust grains, the growth rate of the instability increases. However, the phenomena of excitation of LHW with the help of gyrating ion beam in the presence of negative ions is explained by Sharma *et al.*[15] where the authors observed that the unstable mode frequencies of both the modes increase, with the relative density of negative ions. Moreover, the growth rate of both the unstable modes also increases with relative density of negative ions. Idehara and Tomita[20] produced an ion beam-plasma system in which they vary the beam parameters independently from those of the plasma. They observed that a test wave becomes unstable near the lower-hybrid frequency, and the unstable wave can be seen only where beam is present. The parametric decay instability of lower hybrid waveis studied by Ott[27], which is a significant process in lower hybrid resonance heating phenomenon. The growth rate and thresholds of the instability in inhomogeneous as well as homogeneous plasmas are explained in details. Further, Sharma and Parashar[29] explained the parametric instability of a LHW in a dusty plasma and observed that the dust acoustic mode is strongly modified in the presence of a lower hybrid wave having large amplitude. Lower hybrid wave driven modulation instability in a plasma slab is reported by Konar *et al.*[30]. Here, the authors have found that the modulation instability can be created under the condition $\omega > \omega_{pi}$. Here, we can see that a lot of theoretical and experimental statistics is present to study the lower hybrid waves in different plasma environment but no observations are present to learn the LHW instability in unmagnetized plasma containing multi-ion species. Therefore, in the present work, we study the LHW instability in two-positive ion component unmagnetized plasma. The physics of the paper can be explained as follows: an ion beam with velocity $\vec{v} = v_{b0}\hat{z}$ propagates through one end of the plasma cylinder containingelectrons, light K^+ positive and heavy Cs^+ positive ions that excite the lower hybrid wave instability via Cerenkov interaction[14]. The presence of these positive ions into plasma makes it unstable hence LHW instability is created which is increased with the concentration of positive ions. The unstable modes are analysed in section 1 with the help of fluid theory. In sec. 2 the expression of unstable frequency and growth rate for light positive and heavy positive ion modes are derived. The results and discussion part is presented in Sec. 3. Finally, the conclusion part is provided in Sec. 4.

2 Instability Analysis

Consider a cylindrical magnetized plasma column of radius b_1 which contains electrons, light K^+ positive & heavy Cs^+ positiveion species having densities as $n^{0\,e} = n^{0\,p}$, and $n^{0h+} = \alpha_h\, n^{0\,p}$respectively. Here, α_l and $\alpha_h (= 1 - \alpha_l)$ represents the fractional concentration of light and heavy positive ion species and n^{0p} is the electron plasma density. The mass, charge and temperature of the electrons is taken as: $m^e, -e, T^e$, light positive ions: m^l, e, T^l& heavy positive ions: m^h, e, T^h as respectively,where $T^l \sim T^e \sim T^h$. Consider an ion beam with velocity $\vec{v} = v_{b0}\hat{z}$, mass m^b, density $n^{0b}$and radius $r_{b0} = 2.0$ cm propagates through one end of the plasma cylinder of radius b_1 along the magnetic field $B_s \parallel \hat{z}$.

Before the perturbation, the plasma beam system was almost neutral, such that $-n^{0e} + n^{0l+} + n^{0h+} + n^{0b}$ is approximately zero. Since, there is no electric field existing in the plasma so the plasma equilibrium is disturbed (Fig. 1) by an electrostatic perturbation having potential

$$\phi_1 = \phi(r)e^{-i(\omega\, t - l\,\theta - k_z\, z)} \quad \ldots(1)$$

Since we have taken$n^{0p} >> n^{0b}$, so all the three species i.e., electrons, light K^+& heavy Cs^+ positive ions are treated as fluids defined by equation of motion and equation of continuity as,

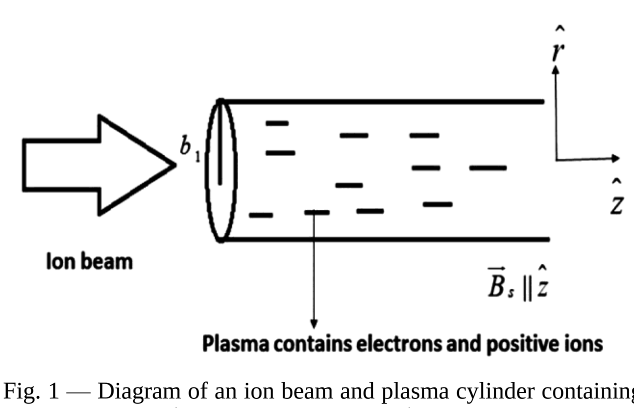

Fig. 1 — Diagram of an ion beam and plasma cylinder containing electrons, light K^+ positive and heavy Cs^+ positive ion species

$$m^e\left[\frac{\partial \vec{v}_E}{\partial t}+\ (\vec{v}_E.\nabla)\vec{v}_E\right]=-e\vec{E}-$$

$$\frac{e}{c}(\vec{v}_E\times B_s)-T^e\frac{\nabla n^e}{n^{0e}} \quad ...(2)$$

$$m^l\left[\frac{\partial \vec{v}_L}{\partial t}+(\vec{v}_L.\nabla)\vec{v}_L\right]=-e\vec{E}-\frac{e}{c}(\vec{v}_L\times B_s)-T^l\frac{\nabla n^l}{n^{0l+}} \quad ...(3)$$

$$m^h\left[\frac{\partial \vec{v}_H}{\partial t}+(\vec{v}_H.\nabla)\vec{v}_H\right]=-e\vec{E}-\frac{e}{c}(\vec{v}_H\times B_s)-T^h\frac{\nabla n^{h+}}{n^{0h+}} \quad ...(4)$$

$$\left[\frac{\partial n^e}{\partial t}+\nabla.(n^e\vec{v}_E)\right]=0, \quad ...(5)$$

$$\left[\frac{\partial n^{l+}}{\partial t}+\nabla.(n^{l+}\vec{v}_L)\right]=0, \quad ...(6)$$

$$\left[\frac{\partial n^{h+}}{\partial t}+\nabla.(n^{h+}\vec{v}_H)\right]=0. \quad ...(7)$$

By using linearization procedure in Eq. (2), we obtain the density perturbation of electrons

$$n^{1e}=-\frac{n^{0e}e\phi_1}{m^e}\left[\frac{k_x^2}{\omega^2-\omega_{ce}^2}+\frac{k_z^2}{\omega^2}\right], \quad ...(8)$$

Using linearization procedure, Eq. (3) gives the perturbed velocity of electrons for the axial and perpendicular case as,

$$v^{1\ z}=-\frac{k_z e\phi_1}{m^e\omega} \quad ...(9)$$

$$v^{\perp\ 1}=\frac{e\,k_x\,\omega\,\phi_1}{m^e(\omega^2-\omega_{ce}^2)}, \quad ...(10)$$

where,

$\omega_{ce}=\frac{eB_s}{m^e\ c}$ represents the electron cyclotron frequency.

Using continuity equation, we get the perturbed density for light positive ions

$$n^{1l+}=\frac{n^{0p}\alpha_l\,e}{m^l}\left[\frac{k_x^2\,\phi_1}{\omega^2-\omega_{c\,l+}^2}+\frac{k_z^2\phi_1}{\omega^2}\right] \quad ...(11)$$

Similarly, for heavy positive ions, the perturbed density is

$$n^{1h+}=\frac{n^{0p}\alpha_h\,e}{m^h}\left[\frac{k_x^2\,\phi_1}{\omega^2-\omega_{c\,h+}^2}+\frac{k_z^2\phi_1}{\omega^2}\right], \quad ...(12)$$

where, $\omega_{ch+}\left(=eB_s/m^h c\right)$ and $\omega_{cl+}\left(eB_s/m^l c\right)$ represents the cyclotron frequency for heavy and light positive ions respectively.

The perturbed density for an ion beam is

$$n^{1b}=-\frac{n^{b0}e\,k_z^2\,\phi_1}{m^b\,(\omega-k_z v_{b0})^2}. \quad ...(13)$$

After putting the values from in the Poisson's equation, we obtain

$$\nabla^2\phi_1=4\pi e\,[n^{1e}-n^{1\,L+}-n^{1\,H+}-n^{1\,b}] \quad ...(14)$$

and using the values, and $\omega_{ph}^2=\frac{4\pi n^{0p}\alpha_h e^2}{m^h}$ for the axially symmetric case we get a 2nd order differential equation in term of ϕ_1

$$\frac{\partial^2\phi_1}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_1}{\partial r}+R^2\phi_1=-\ \frac{\omega_{pb}^2\ k_z^2\ \phi_1}{(\omega-k_z v_{b0})^2\,P}. \quad ...(15)$$

where,

$$P=\left(1-\frac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2}-\frac{\alpha^l\omega_{pl}^2}{\omega^2-\omega_{c\,l+}^2}-\frac{\alpha^h\omega_{ph}^2}{\omega^2-\omega_{c\,h+}^2}\right),$$

$$R^2=\left(\frac{\frac{\omega_{pe}^2\ k_z^2}{\omega^2}+\frac{\omega_{pl}^2\ k_z^2\alpha^l}{\omega^2}-k_z^2+\frac{\omega_{p\,h}^2\ k_z^2\alpha^h}{\omega^2}}{1-\frac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2}-\frac{\alpha^l\omega_{pl}^2}{\omega^2-\omega_{c\,l+}^2}-\frac{\alpha^h\omega_{ph}^2}{\omega^2-\omega_{c\,h+}^2}}\right).$$

Therefore, the above Eq. (15) can be rewritten as

$$\nabla_\perp^2\phi_1+R^2\phi_1=-\ \frac{\omega_{pb}^2\ k_z^2\ \phi_1}{(\omega-k_z v_{b0})^2\,P}. \quad ...(16)$$

Eq. (16), in the absence of ion beam, can be simplified as

$$\frac{\partial^2\phi_1}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_1}{\partial r}+\left(p_m^2-\frac{l^2}{r^2}\right)\phi_1=0 \quad ...(17)$$

Here, Eq. (17) is a Bessel equation having solution $\phi_1=AJ_l(p_m\,r)$, at $r=b_1$,

$\phi_1=0\Rightarrow\ p_m=X_n/b_1\ \ (\ n=1,2,3,4\ldots\ldots\ldots)$

i.e., X_n represents the zeros of the Bessel function $J_0(X)$. The wave function ϕ_1 in the existence of ion beam can be expressed as

$$\phi_1=\sum_m A_m\ J_0(p_m\,r). \quad ...(18)$$

Now, put the value of ϕ_1 in Eq. (17), from Eq. (18) and multiplying both sides of Eq. (17) by $r\,J_0(p_n\,r)$& integrate from 0 to b_1 and by using Dirac Delta property, we obtain

$$R^2-p_m^2=-\ \frac{\omega_{pb}^2\ k_z^2\ \phi_1}{(\omega-k_z v_{b0})^2\ P}. \quad ...(19)$$

Upon entering all the values, the aforementioned Eq. (19) can be expressed as

$$-(k_z^2\ +p_m^2)+\frac{\omega_{p\,e}^2\,k_z^2}{\omega^2}+\frac{\omega_{p\,l}^2\,k_z^2\alpha_l}{\omega^2}+\frac{\omega_{p\,h}^2\,k_z^2\alpha_h}{\omega^2}+$$
$$\frac{\omega_{pe}^2\ p_m^2}{\omega^2-\omega_{ce}^2}+\frac{\omega_{p\,l}^2\,\alpha_l p_m^2}{\omega^2-\omega_{c\,l+}^2}+\frac{\omega_{p\,h}^2\,\alpha_h p_m^2}{\omega^2-\omega_{c\,h+}^2}=-\ \frac{\omega_{pb}^2\ k_z^2\ \phi_1}{(\omega-k_z v_{b0})^2\ \mathrm{P}} \quad ...(20)$$

Dividing both sides of Eq. (20) by $-(k_z^2+p_m^2)$, we obtained

$$1-\frac{\omega_{p\,e}^2\ k_z^2}{\omega^2(k_z^2\ +p_m^2)}-\frac{\omega_{p\,l}^2\ k_z^2\alpha_l}{\omega^2(k_z^2\ +p_m^2)}-$$
$$\frac{\omega_{p\,h}^2\ k_z^2\alpha_h}{\omega^2(k_z^2\ +p_m^2)}-\frac{\omega_{pe}^2\ p_m^2}{\omega^2-\omega_{ce}^2(k_z^2\ +p_m^2)}-$$

$$\frac{\omega_{p\,l}^2\,\alpha_l p_m^2}{(k_z^2+p_m^2)} - \frac{\omega_{p\,h}^2\,\alpha_h p_m^2}{\omega^2-\omega_{c\,h+}^2(k_z^2+p_m^2)} = \frac{\omega_{pb}^2\,k_z^2\,\phi_1}{(\omega-k_z v_{b0})^2\,(k_z^2+p_m^2)P}. \quad \ldots(21)$$

Now, Eq.(21) is simplified, and both sides are multiplied by $\frac{(k_z^2+p_m^2)}{p_m^2}$, also assuming $\alpha = \frac{(k_z^2+p_m^2)}{p_m^2} + \frac{\omega_{pe}^2}{\omega_{ce}^2}$, then Eq. (21) can be written as:

$$1 - \frac{\omega_{pe}^2\,k_z^2}{\omega^2 p_m^2\,\alpha} - \frac{\omega_{p\,l}^2\,k_z^2\alpha_l}{\alpha\,\omega^2 p_m^2} - \frac{\omega_{p\,l}^2\,\alpha_l}{\alpha\,(\omega^2-\omega_{c\,l+}^2)} - \frac{\omega_{p\,h}^2 k_z^2\alpha_h}{\alpha\,\omega^2 p_m^2} - \frac{\omega_{ph}^2\,\alpha_h}{\alpha\,(\omega^2-\omega_{ch+}^2)} = -\frac{\omega_{pb}^2\,k_z^2}{\alpha\,(\omega-k_z v_{b0})^2\,p_m^2} \quad \ldots(22)$$

2.1 Growth Rate in the Existence of Light Positive Ions

In the existence of light positive potassium ions, Eq. (22) can be revised as

$$1 - \frac{\omega_{pe}^2\,k_z^2}{\omega^2\,p_m^2\,\alpha} - \frac{\omega_{p\,l}^2\,k_z^2\alpha_l}{\alpha\,\omega^2 p_m^2} - \frac{\omega_{p\,l}^2\,\alpha_l}{\alpha\,(\omega^2-\omega_{cl+}^2)} = \frac{\omega_{pb}^2\,k_z^2}{\alpha\,(\omega-k_z v_{b0})^2\,p_m^2} \quad \ldots(23)$$

Multiplying both sides by $\omega^2(\omega^2-\omega_{cl+}^2)$, Eq. (23) can be written as

$$\omega^2(\omega^2-\omega_{cl+}^2) - \frac{\omega^2(\omega^2-\omega_{cl+}^2)\omega_{pe}^2\,k_z^2}{\omega^2\,p_m^2\,\alpha} - \frac{\omega^2(\omega^2-\omega_{cl+}^2)\omega_{p\,l}^2\,k_z^2\alpha_l}{\alpha\,\omega^2 p_m^2} - \frac{\omega^2(\omega^2-\omega_{cl+}^2)\,\omega_{p\,l}^2\,\alpha_l}{\alpha\,(\omega^2-\omega_{cl+}^2)} = \frac{\omega^2(\omega^2-\omega_{cl+}^2)\omega_{pb}^2\,k_z^2}{\alpha\,(\omega-k_z v_{b0})^2\,p_m^2} \quad \ldots(23)$$

and after rearranging the terms in above equation, we obtain

$$\omega^4 - \omega^2\left(\omega_{c\,l+}^2 + \frac{\omega_{pe}^2\,k_z^2}{\alpha\,p_m^2} + \frac{\omega_{c\,l+}^2\,\alpha_l}{\alpha} + \frac{\omega_{pl}^2\alpha_l\,k_z^2}{\alpha\,p_m^2}\right) + \frac{\omega_{c\,l+}^2\,\omega_{pe}^2\,k_z^2}{\alpha p_m^2} + \frac{\omega_{pl}^2\,\alpha_l k_z^2\omega_{cl+}^2}{\alpha\,p_m^2} = -\frac{\omega_{pb}^2\,k_z^2\,\omega^2(\omega^2-\omega_{cl+}^2)}{\alpha\,(\omega-k_z v_{b0})^2\,p_m^2}. \quad \ldots(24)$$

Equation (24) can be expressed as

$$(\omega^2-c_1^2)(\omega^2-c_2^2)(\omega-k_z v_{b0})^2 = \frac{\omega^2(\omega^2-\omega_{cl+}^2)\omega_{pb}^2\,k_z^2}{\alpha\,p_m^2}, \quad \ldots(25)$$

where,

$$c_1^2 = \omega_{cl+}^2 + \frac{\omega_{lh}^2\,k^2\,\alpha_l}{p_m^2}\left(1+\frac{n^{0p}\,m^{l+}k_z^2}{m^e\,n^{0l+}\,\alpha_l\,k^2}\right). \quad \ldots(26)$$

$$c_2^2 = \frac{\frac{\omega_{lh}^2\,\omega_{cl+}^2\alpha_l\,k_z^2}{p_m^2}\left(1+\frac{\omega_{pe}^2}{\omega_{pl}^2\,\alpha_l}\right)}{\omega_{cl+}^2 + \frac{\omega_{lh}^2\,\alpha_l k^2}{p_m^2}\left(1+\frac{k_z^2\,n^{0p}\,m^{l+}}{k^2\,m^e\,n^{0l+}\,\alpha_l}\right)}.$$

Here, assume $\alpha \approx 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}$.

Also, $\omega \approx c_1$ resembles to the light (K^+) ion mode and $\omega \approx k_z v_{0b}$ consistent to the ion beam mode. But, we are seeing for solutions when $\omega \approx k_z v_{0b}$, and the two factors present in the L.H.S. of Eq. (25) are instantaneously zero in the lack of ion beam but in the existence of ion beam, we can expand ω as $\omega \approx c_1 + \delta_1 \simeq k_z v_{0b} + \delta_1$, where δ_1 is the minor frequency discrepancy due to finite R.H.S of Eq. (25). Then Eq. (25) provides the growth rate of light positive ions mode:

$$\Gamma = \mathrm{Im}(\delta_1) = \left[\frac{c_1}{2}\frac{(c_1^2-\omega_{cl+}^2)\,\omega_{pb}^2\,k_z^2}{p_m^2\,(\omega^2-c_2^2)}\right]^{1/3}\frac{\sqrt{3}}{2} \quad \ldots(27)$$

The unstable real wave frequency for light K^+ positive ion in terms of beam energy is given by

$$\omega_r = k_z\left(\frac{2eV_b}{m^b}\right)^{1/2} - \frac{1}{2}\left[\frac{c_1}{2}\frac{(c_1^2-\omega_{cl+}^2)\,\omega_{pb}^2\,k_z^2}{p_m^2\,(\omega^2-c_2^2)}\right]^{1/3}, \quad \ldots(28)$$

where, eV_b represent the beam energy.

2.2 Growth Rate in the Existence of Heavy Positive Ions

However, in the absence of light positive ions, Eq. (21) can be modified as

$$1 - \frac{\omega_{pe}^2\,k_z^2}{\omega^2\,p_m^2\,\alpha} - \frac{\omega_{p\,h}^2\,k_z^2\alpha^h}{\alpha\,\omega^2 p_m^2} - \frac{\omega_{ph}^2\,\alpha^h}{\alpha\,(\omega^2-\omega_{ch+}^2)} = \frac{\omega_{pb}^2\,k_z^2}{\alpha\,(\omega-k_z v_{b0})^2\,p_m^2}. \quad \ldots(29)$$

Multiplying Eq. (29), on both sides by $\omega^2(\omega^2 - \omega ch{+}2$ and after simplifying the equation we obtain

$$\omega^4 - \omega^2\left(\omega_{c\,h+}^2 + \frac{\omega_{pe}^2\,k_z^2}{\alpha\,p_m^2} + \frac{\omega_{c\,h+}^2\,\alpha_h}{\alpha} + \frac{\omega_{ph}^2\alpha_h\,k_z^2}{\alpha\,p_m^2}\right) + \frac{\omega_{c\,h+}^2\,\omega_{pe}^2\,k_z^2}{\alpha p_m^2} + \frac{\omega_{ph}^2\,\alpha_h k_z^2\omega_{ch+}^2}{\alpha\,p_m^2} = -\frac{\omega_{pb}^2\,k_z^2\,\omega^2(\omega^2-\omega_{ch+}^2)}{\alpha\,(\omega-k_z v_{b0})^2\,p_m^2}. \quad \ldots(30)$$

The above equation can be rewritten as

$$(\omega^2-d_1^2)(\omega^2-d_2^2)(\omega-k_z v_{b0})^2 = \frac{\omega^2(\omega^2-\omega_{ch+}^2)\omega_{pb}^2\,k_z^2}{\alpha\,p_m^2}. \quad \ldots(31)$$

Where

$$d_1^2 = \omega_{ch+}^2 + \frac{\omega_{lh}^2 \alpha_h k^2}{p_m^2}\left(1 + \frac{k_z^2 n^{0p} m^{h+}}{k^2 m^e n^{0h+} \alpha_h}\right) \quad \ldots(32)$$

and $$d_2^2 = \frac{\frac{\omega_{lh}^2 \omega_{ch+}^2 \alpha_h k_z^2}{p_m^2}\left(1+\frac{\omega_{pe}^2}{\omega_{ph}^2 \alpha^h}\right)}{\omega_{ch+}^2+\frac{\omega_{lh}^2 \alpha_h k^2}{p_m^2}\left(1+\frac{k_z^2 n^{0p} m^{h+}}{k^2 m^e n^{0h+} \alpha_h}\right)}$$

Here, $\omega \approx d_1$ resembles to the heavy (Cs^+) positive ion mode and $\omega \approx k_z v_{0b}$ consistent to the ion beam mode. But, we are seeing for results when $\omega \approx k_z v_{0b}$ and the two factors present on the L.H.S. of Eq. (31) are instantaneously zero in the lack of ion beam but in the existence of ion beam, we can expand ω as $\omega \approx d_1 + \delta_1 \simeq k_z v_{0b} + \delta_1$, where δ_1 is the minor frequency discrepancy due to finite R. H.S. of Eq. (31). Then Eq. (31) provides the growth rate of Cs^+ positive ions mode as

$$\Gamma = \text{Im}(\delta_1)\left[\frac{d_1}{2}\frac{(d_1^2-\omega_{ch+}^2)\,\omega_{pb}^2\,k_z^2}{p_m^2\,(\omega^2-d_2^2)}\right]^{1/3} \frac{\sqrt{3}}{2} \quad \ldots(33)$$

The unstable real wave frequency for heavy (Cs^+) positive ion mode in terms of ion beam energy is given by

$$\omega_r = k_z\left(\frac{2eV_b}{m^b}\right)^{1/2} - \frac{1}{2}\left[\frac{d_1}{2}\frac{(d_1^2-\omega_{ch+}^2)\,\omega_{pb}^2\,k_z^2}{p_m^2\,(\omega^2-d_2^2)}\right]^{1/3}$$

Where, $\omega_{pb} = \left(\frac{4\pi n^{b0} e^2}{m^b}\right)^{1/2}$ represents the plasma frequency of an ion beam.

3 Results and Discussion

The plasma parameters[14] that are being used in the present study on lower hybrid waves are as follows: electron plasma density $n^{0e} = n^{0p} = 10^9 cm^{-3}$, $T^e \approx T^L \approx T^H$ =0.2eV, radius of plasma column $b_1 = 2cm$, beam radius of potassium beam, $r_{b0} = 2\,cm$, beam energy E_b=10eV, guiding magnetic field $B_s \approx 3\times10^3$ gauss. Here, we observed that with the increase in relative concentration of positive ions, the lower hybrid wave exhibit two modes, a light K^+ mode and a heavy Cs^+ mode. The frequencies $\omega\,(rad/\text{sec})$ and the wave numbers $k_z(cm^{-1})$ of the unstable wave modes can be calculated by the point of intersection of the ion beam mode and the light or heavy positive ions plasma modes. Using Eq. (25) in Fig. 2, the frequency $\omega\left(\frac{rad}{\text{sec}}\right)$ versus wave number $k_z(cm^{-1})$ in the presence of light K^+ positive ion for different relative concentration α_l of light positive ions is plotted and we observed that with increasing the wave number, frequency increases.

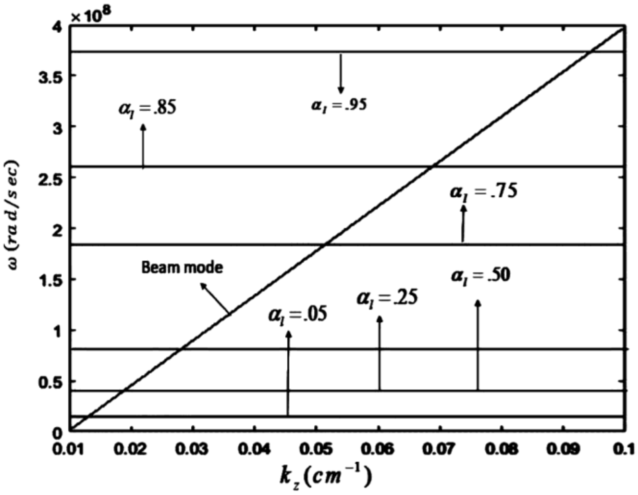

Fig. 2 — Dispersion relation of longitudinal electrostatic LHW in the existence of light K^+ positive ion species and a beam mode

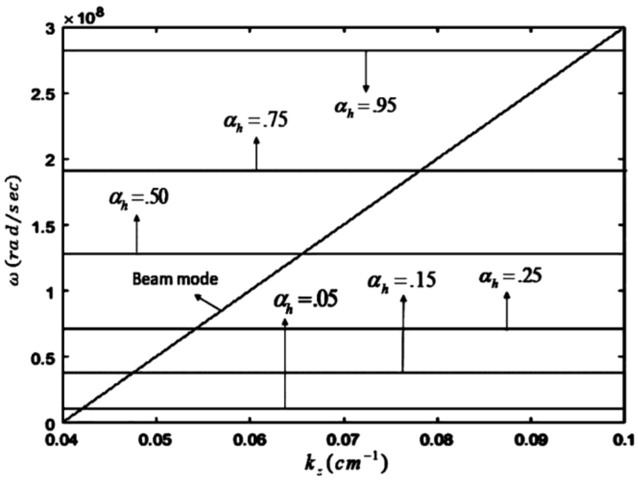

Fig. 3 — Dispersion relation of longitudinal electrostatic LHW in the existence of heavy Cs^+ positive ion species and a beam mode

In Fig. 3, the frequency $\omega\,(rad/\text{sec})$ versus wave number $k_z(cm^{-1})$ is plotted for different relative concentrations α_h of heavy Cs^+ positive ions. The theoretical work presented here is qualitatively similar with the experimental observations of Idehara Tomita[20], as the trends of the graphs [Fig. 3] are similar [cf. fig.10 of the reference paper].Plotting the growth rate against the fractional concentration of light positive ions using Eq. (27) confirms the following result: In Fig. 4, the growth rate of instability increases with the (α_l) relative concentration of light positive ions. This is because; adding ions to the plasma in turn increases the LHW instability, which in turn causes the growth rate to rise.

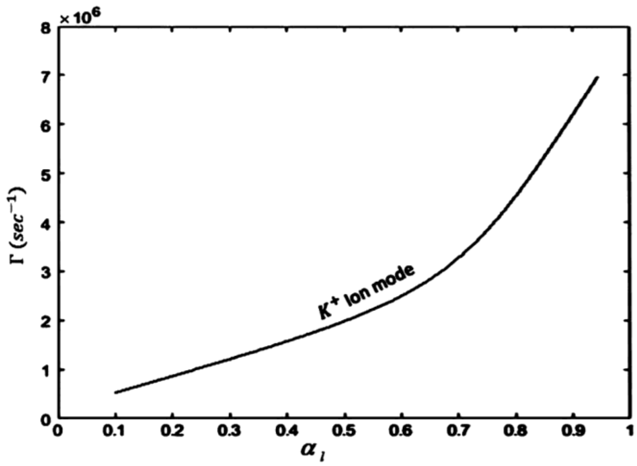

Fig. 4 — Growth rate Γ ($\sec^{-1}$) of the unstable light positive K^{+} ion mode as a function of fractional concentration α_l of light positive ion species

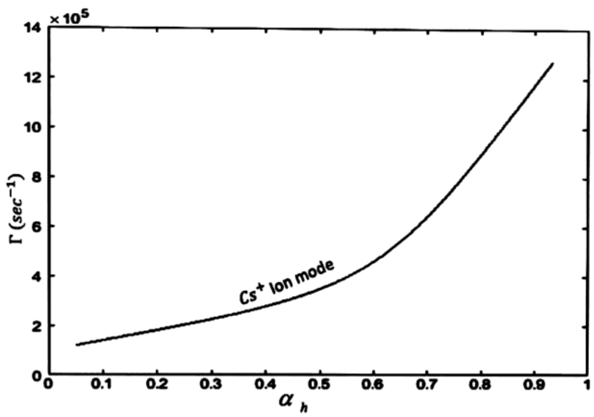

Fig. 5 — Growth rate Γ($\sec^{-1}$) of the unstable heavy positive Cs^{+} ion mode as a function of fractional concentration α_h of heavy positive ion species

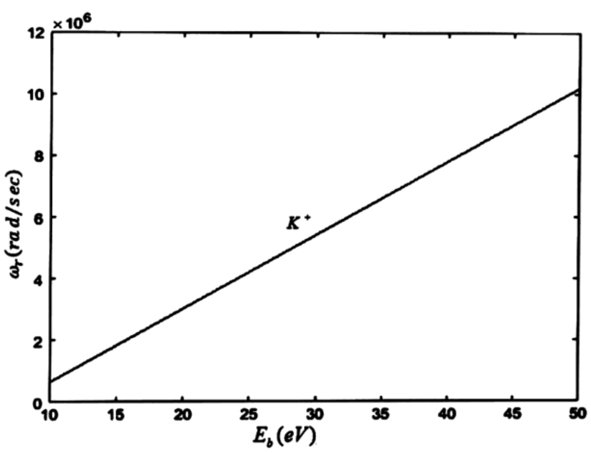

Fig. 6 — Unstable real wave frequency as a function of beam energy for light K^{+} positive ion

The growth rate versus relative concentration of heavy positive ions is plotted in Figure 5, and we can

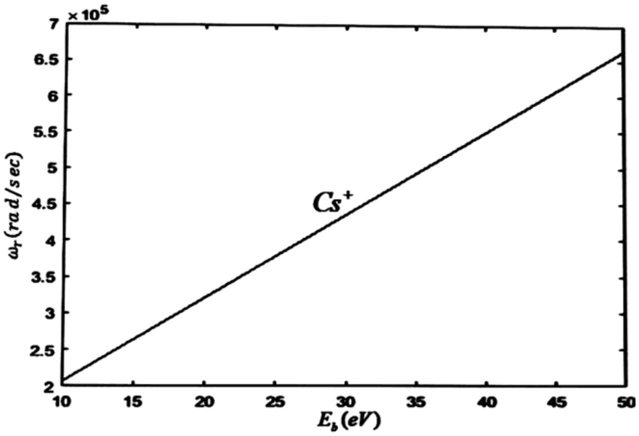

Fig. 7 — Unstable real wave frequency as a function of beam energy for heavy Cs^{+} positive ion

see that the growth rate $\Gamma(\sec^{-1})$grows when the relative concentration α_h of heavy positive ions. However, as compared to the heavy Cs^{+} positive ion mode, the growth rate rise is greater for the light K^{+} positive ion mode. In Fig. 6, the unstable (ω_r) real wave frequency versus beam energy E_b is plotted and we observed that with increasing the beam energy the unstable real wave frequency increases. It can be understood as follows: if the ion beam is having high energy, then it interacts with plasma at large velocity due to which instability will increase.

Using Eq.(34), the unstable real wave frequency versus ion beam energy in the existence of heavy Cs^{+} positive ions is plotted in Fig. 7 and we observed that with increasing the ion beam energy the unstable wave frequency increases.

4 Conclusion

It can be concluded that an increase in the fractional concentration of positive ions in plasma causes the (LHW) lower hybrid wave instability in a magnetized collisionless plasma to be excited. The LHW is then excited into two unstable modes(light K^{+} ion and heavy Cs^{+}ion) by an ion beam passing through the collisionless magnetized plasma, which is powered by the Cerenkov interaction. The unstable real wave frequencies & the growth rate of both the modes are found to increase with the fractional concentrations of the corresponding ion species. However, this increase is more rapid in case of light positive ions and we have also observed the important impact of increase in beam energy on the unstable real wave frequency for both the light & heavy unstable modes. We found that with increasing the beam energy, the instability will grow up, so unstable wave frequency increases. The earth's ionosphere, solar wind, and interstellar nebulae, which are made up of a mixture of several positive ion species are sources of positive ion plasma, e.g., H^+, O^+, O_2^+ & $NO^+ etc.$, whereas the concentration of these ions rise with height. Therefore, the theory present in our work may be useful in the understanding of the many astronomical phenomenon. Also, lower hybrid current drive is a proficient method to drive non-inductive current in tokamak plasmas. So, the present study can be helpful in understanding the LHWs and how they can be affected by the tokamak plasma components which in turn can affect the performance of a tokamak.

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