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Generation of Higher Order Laguerre-Gaussian Beams through Combination of Lower Order Phase Elements

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Laguerre-Gaussian (LG) beam is a vortex beam, which is characterised by its helical phase, and has a number of potential applications apart from being a precursor for generation of other vortex beams. Generation of LG beams, carrying higher-order orbital angular momentum (OAM), have been demonstrated by the utilization of multiple vortex phase plates (VPPs) with capability of imparting lower topological charges (TCs). The TCs of the generated LG beams were experimentally ascertained by the analysis of fork patterns generated by interfering the generated LG beams with their corresponding reference Gaussian beams. The experimentally obtained amplitude profiles and fork patterns, exhibited a fair agreement with the results of theoretical formulations.

Keywords: L G beams; Orbital angular momentum; Topological charges; Graphene

1 Introduction

The concept of angular momentum of light was first proposed by R. A. Beth in 1936[1]. The experimental results indicate that assigning an angular momentum of $\hbar$ ($-\hbar$) to each quantum of left (or right) circularly polarized light in a vacuum consistently produces the same quantitative outcome. This assumption is based on the conservation of angular momentum at the interface of the plate.. However, Allen et al. demonstrated that photons in a vortex beam carry OAM of l under paraxial conditions[2], leading to widespread study and application of vortex beams.

LG beams have been one of the most intriguing vortex beams because of their unique properties, making them a potential candidate for a broad range of applications[3]. LG beams are characterized by intricate spatial and phase profiles. These beams are solutions of the paraxial wave equation in cylindrical coordinates, in which each photon possesses a well-defined OAM quantized in unit of Planck′s constant[3,4]. The radial distribution is governed by Laguerre polynomials, which give rise to concentric rings of varying intensity. Meanwhile, the azimuthal dependence leads to helical phase fronts, providing LG beams its distinctive twist carrying OAM. The LG beam carries a quantized amount of OAM, with the number of helical wave fronts, in one wavelength, determining its TC. This enables the beams to carry information not only in their intensity and phase but also in their OAM, paving the way for their application in communication and manipulation techniques[5]. The major advantage of these vortex beams in optical communication arises from the infinite degree of freedom, that could be provided by the different OAMs being carried by a vortex beam[6]. Advanced optical setups, such as spiral phase plates, spatial light modulators, and diffractive optical elements, enable the controlled transformation of Gaussian beams into LG modes[7-9]. Amongst these, the spiral phase plates are preferred, owing to the flexibility provided by them in optical setup, especially required for complex applications like optical communication[10]. However, the use of spiral phase plate is limited by the restriction in attaining higher-order OAM states, which in turn limits the data-handling capability of the communication channel. Recently, Imbert-Fedorov (IF) shift of LG beams in a graphene-VO_2 structure has been studied[11].

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In the present manuscript, a simple technique has been used to achieve higher order LG beams by combining multiple spiral phase plates. The fork patterns have been generated using interference technique for ascertaining the TCs of the generated LG beams[12]. The results have been supported by theoretical formulations. The technique could be used in generating higher order vortex beams which would increase the data handling capacities of optical communication channel.

2 Theory

LG beams are the symmetrical solution of paraxial wave equation in circular cylindrical coordinates (r, θ, z), and are given by the equation[11],

$$LG_p^l = \sqrt{\frac{2p!}{\pi(p+|l|)!}}\left(\frac{1}{w(z)}\right)\left[\frac{\sqrt{2}r}{w(z)}\right]^{|l|} exp\left(-\frac{r^2}{w^2(z)}\right) L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right) exp\left[\frac{ikr^2z}{2(z^2+z_r^2)} - il\theta - i(2p+|l|+1)tan^{-1}\left(\frac{z}{z_r}\right)\right] \quad \ldots (1)$$

where, $w(z) = w_0\sqrt{1+\frac{z^2}{z_R^2}}$ represents the beam radius at z, while w_0 is the beam radius at $z = 0$ referred to as the beam waist. Additionally, $z_r = \frac{\pi w_0}{\lambda}$ denotes the Rayleigh range of the Gaussian beam, $k = \frac{2\pi}{\lambda}$ is the wave number and $L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right)$ is the generalised Laguerre polynomial with argument $\left(\frac{2r^2}{w^2(z)}\right)$. $R = z\left[1+\left(\frac{z_r}{z}\right)^2\right] = \frac{(z^2+z_r^2)}{z}$ represents the radius of curvature and $\phi = (2p+|l|+1)tan^{-1}\left(\frac{z}{z_r}\right)$ signifies the Gouy Phase which changes with the propagation distance from waist position.

For simplicity, taking the radial index, $p = 0$, the equation for LG beam can be given by:

$$LG_0^l = \sqrt{\frac{2!}{\pi|l|!}}\left(\frac{1}{w(z)}\right)\left[\frac{\sqrt{2}r}{w(z)}\right]^{|l|} exp\left(-\frac{r^2}{w^2(z)}\right) exp\left[\frac{ikr^2}{2R} - il\theta - i\phi\right] \quad \ldots (2)$$

The term $exp\left(-\frac{r^2}{w^2(z)}\right)$ describes a general Gaussian beam. The term $exp(-il\theta)$ represents the phase of the optical ortex, which cancels out in the centre of the vortex due to destructive interference giving rise to the singularity at the centre which manifests as the central-darkness. This is responsible for giving the donut like shape of the intensity profile.

Considering a LG beam ($LG_0^{l_1}$) carrying TC l_1, passing through a spiral phase element of TC l_2, would give rise to a LG beam LG_0^l. Then, LG_0^l would be given by

$$LG_0^l = LG_0^{l_1} * LG_0^{l_2} \quad \ldots (3)$$

where, $LG_0^{l_2}$ is a LG beam carrying the TC of l_2. This emulates a Gaussian beam gaining spiral phases as it passes through two spiral phase elements having TCs l_1 and l_2.

The TC of the generated LG beams have been ascertained by producing fork patterns, obtained by their interference with reference Gaussian beam[12]. In order to theoretically study the effects on the phase of resultant LG beam, the resultant LG beam is superposed with the Gaussian beam as: '

$$LG_0^l = LG_0^{l_1} * LG_0^{l_2} * G \quad \ldots (4)$$

The superposition yields a distinctive fork pattern corresponding to the topological charge of the resultant LG beam.

3 Experimental Setup

The schematic diagram of the experimental setup is shown in Fig. 1. A Gaussian beam, from a He-Ne laser of appropriate intensity, controlled through neutral density filter, is collimated using a set of two lenses. Two identical Gaussian beams are obtained using a 50/50 beam-splitter. One of these beams is used to generate LG beam, while the other beam is used as the reference. The LG beams are obtained by passing the laser beam through multiple VPPs having different combinations of TCs. The VPPs used in the experiment could provide a TC of at most 8. The generated LG beam is made to interfere with the reference Gaussian beam using a second beam splitter, to ascertain the charge carried by the vortex beam. A CCD camera is used to capture the images.

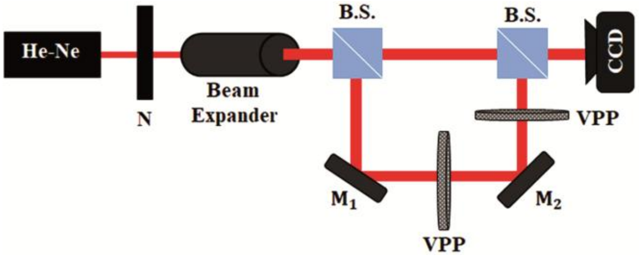

Fig. 1 — Schematic diagram of the experimental set-up; He-Ne: laser source of wavelength 633 nm; N: Neutral density filter; M$_1$ and M$_2$: Mirrors; VPP: vortex phase plate; B.S.: beam splitter (50/50); CCD: CCD camera.

4 Results and Discussion

LG beams were experimentally obtained by passing Gaussian beam through two VPPs, with different combinations of TC on them. A number of higher order LG beams were generated using different combinations of TCs on the VPPs, followed by the generation of the fork patterns. Remarkably, these experimental results yield amplitude profiles and fork-shaped fringes that closely resemble the patterns obtained through theoretical analyses.

The patterns generated for two different combinations of l_1 and l_2 are shown in Fig. 2. In both the figures the fork patterns for the LG beam carrying TC of l_1 (a) experimental and (d) theoretical) and l_2 (b) experimental and (e) theoretical) individually, are shown along with the higher order LG beam generated through the VPP combination carrying TC of l_1+l_2 (c) experimental and (f) theoretical). The distinctive fork patterns serve as a tool for determining the topological charge of an LG beams, as the number of dark fringes within the fork pattern directly corresponds to the topological charge of the respective beam. Once these fringes are excluded, the remaining dark fringes precisely match the topological charge of the LG beam. The fork patterns shown in Fig. 2 (i) & (ii) (c & f) clearly indicates that LG beams of TCs 10 and 12, are respectively obtained, through the VPP combinations.

It is interesting to point out that beams with l=0 could be obtained by applying equal and opposite TCs to the Gaussian beam (Fig. 2 (iii) and (iv)). It may be observed from the figures that patterns of the LG beams carrying TCs l_1 (parts a, d) and l_2 (parts b, e) individually, indicate same number of fringes within

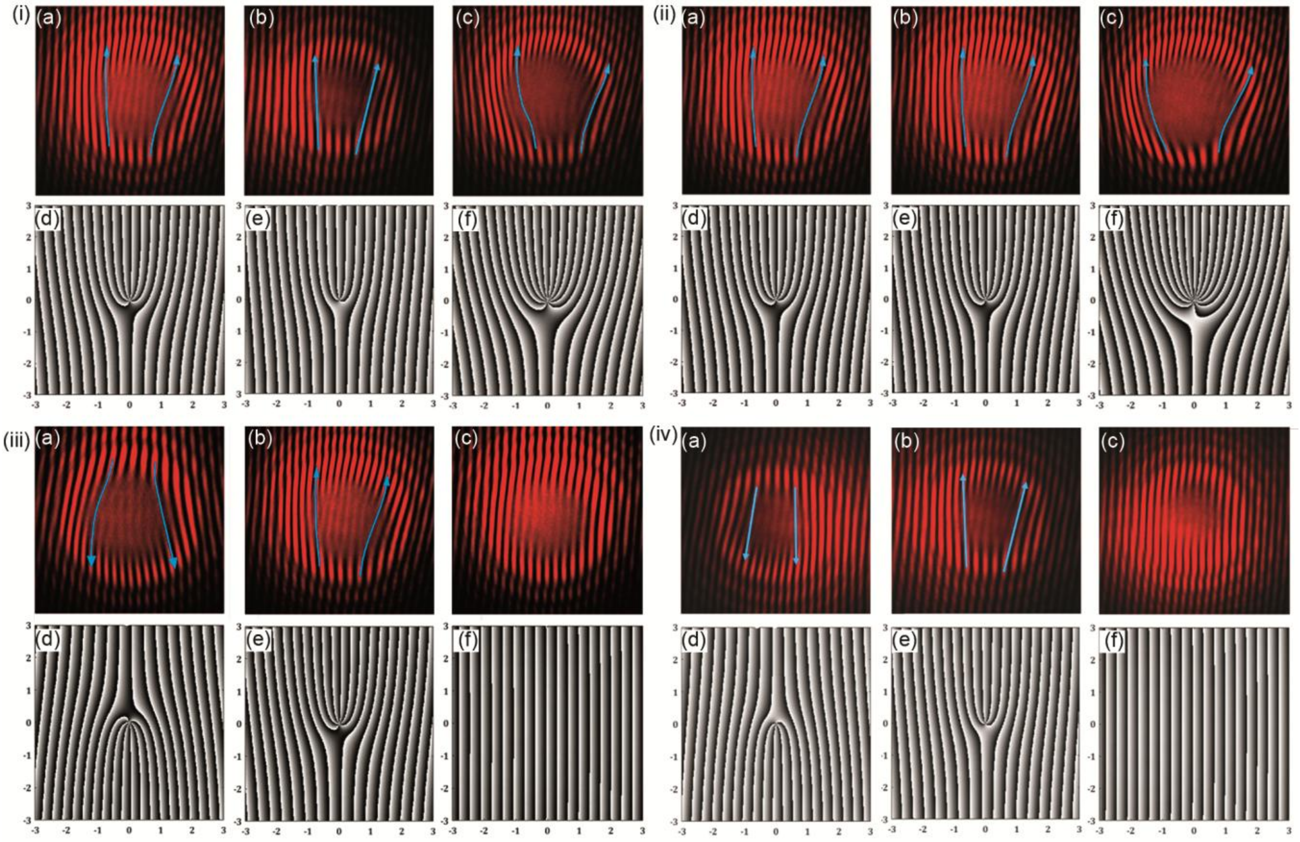

Fig. 2 — (i) Fork patterns for LG beams (generated by individual VPP) of $l_1=6$ ((a) experimental; (d) theoretical) and $l_2=4$ ((b) experimental; (e) theoretical); Fork pattern ((c) experimental; (f) theoretical) for LG beam (generated by combination of VPP) of $l=10$, (ii) Fork patterns for LG beams (generated by individual VPP) of $l_1=6$ ((a) experimental; (d) theoretical) and $l_2=6$ ((b) experimental; (e) theoretical); Fork pattern ((c) experimental; (f) theoretical) for LG beam (generated by combination of VPP) of $l=12$, (iii) Fork patterns for LG beams (generated by individual VPP) of $l_1=-6$ ((a) experimental; (d) theoretical) and $l_2=6$ ((b) experimental; (e) theoretical); Fork pattern ((c) experimental; (f) theoretical) for LG beam (generated by combination of VPP) of $l=0$, (iv) Fork patterns for LG beams (generated by individual VPP) of $l_1=-4$ ((a) experimental; (e) theoretical) and $l_2=4$ ((b) experimental; (f) theoretical); Fork pattern ((c) experimental; (g) theoretical) and amplitude profile ((d) experimental; (h) theoretical) for LG beam (generated by combination of VPP) of $l=0$.

the fork pattern but having opposite orientation, implying that the LG beams carry equal and opposite TCs. However, the patterns obtained by interference of the reference Gaussian beams with the LG beam obtained by the combination of the VPPs (Fig. 2 (iii) (c & f) and 2 (iv) (c & f)), do not show any fork patterns indicating a zero TC.

These results clearly indicate that the LG beam obtained by employing simultaneous combination of VPPs carries a TC which is the sum of the individual charges imparted by the VPPs independently. Thus higher-order LG beams can be generated by using combination of vortex phase plates of lower orders.

5 Conclusion

In conclusion, this experimental study successfully demonstrated the generation of higher order LG beams using multiple VPPs. LG beams of higher orders were experimentally demonstrated by combining the TCs on the individual VPPs. The TCs on the LG beams were ascertained by using the fork patterns obtained through interference of the generated LG beams with the corresponding reference Gaussian beams. The experimental amplitude profiles and the fork patterns corresponded well with the theoretical results. The results show that the combinations of VPPs with different TCs can yield higher order LG beam carrying a net TC equal to the sum of the individual TCs imparted by the VPPs independently. This approach of generating higher-order LG beams by utilizing VPPs of lower orders could pave a path for its potential applications in various fields of optical vortices apart from optical communication.

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