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Reexamination of Staggering in Gamma Band Energy: A Novel Formulation to Understand Shape Transitions in Nuclei

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The study of deformed atomic nuclei focuses on distinguishing between axial γ-rigid and γ-soft asymmetric configurations. In our groundbreaking research, we offer a staggering parameter S(4) as an indicator, linking it with the asymmetry parameter (γ0) and the Grodzins product ($E(2_1^+)$*B(E2)) in distorted nuclei. This work uses empirical data to determine a first-order phase change in nuclei by examining the association between rotational momentum (J=4). This method answers long-standing concerns and is used to well-deformed isotopes (Gd to Hf). This revolutionary method's simplicity suggests that it has a wide range of applications.

Keywords: Asymmetric parameter; Grodzins product; Staggering

1 Introduction

In order to objectively discern between the previously stated potentials, the energy staggering that occurs inside the quasi-band is an important attribute that is frequently accessible through experimental data or that can be evaluated in new nuclei in a short period of time. Throughout history, staggering has been extremely important[1] as an indicator of the reliance of potential. This article uses the term "axially rigid region" to define the region between the vibrator and the axially symmetric rotor. In this region, the symbol γ is stiff and aligned with a harmonic oscillator potential. The lowest gamma value is extremely close to zero. We will show that studying energy staggering along an isotopic chain can be a simple and quick way to determine how the structure of a nuclear atom has evolved over time, and it may even provide some insights into the process by which phase transitions occur. Atomic nuclide configuration is determined by the interaction of a single particle with the total number of degrees of freedom. Conventional wisdom has held that atomic nuclides had globular shapes, despite the fact that extensive studies of their ground and excited states have revealed the presence of non-globular geometries in a number of nuclides. The Bohr and Mottelson model[2] is a widely used technique for characterizing the geometric properties of atomic nuclides. This model relies on two fundamental parameters: γ, which describes the regularity of ellipsoids and runs from 0 to π/3 degrees, and β, which ranges from 0 to 0.3 and reflects the inherent quadrupole moment. The asymmetry variable γ0 indicates that the nuclear matrix has an extended structure that eventually flattens out. This is demonstrated by the nuclear matrix. The identification of whether axially asymmetric atomic nuclei come from a potential with clearly defined minima or softer minima with respect to gamma is a significant barrier to understanding their structural properties. The Davydov model[3] provides an explanation for the previously discussed rigid γ requirement. The Wilets-Jean model[4], which allows γ to freely vary between and and the O(6) limit within the context of the IBA model, can explain the second condition, which is marked by a completely horizontal potential in γ.[5-8] For less severe cases, nuclear models with shallow minima, such as the geometric collective model[9] or the IBA, can be used to calculate softness. These two geometric restrictions, in their current configuration, define the spectrum of axially asymmetric nuclei. The soft and rigid varieties have the same amount of ground-state energy. Despite this, the energy levels of the stimulated bands differ significantly, with both models producing an impressive amount of visible energy. Gupta[10], on the other hand, proposes a novel approach to studying ground-state energy levels. In contrast, Zamfir and Casten[1] found that nuclei with

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strong asymmetry are more likely to be γ-soft than γ-rigid. This was revealed during their examination into staggering indexes. The DF model is commonly used to describe the behaviour of nuclides positioned between virtually periodic and rotating forms of collective excitation. As a result of Davidson[11] construction of a link between the ARM and the rotational vibration (RV) framework, it was observed that the two models provided equally positive results. Davydov and Rostovsky studied β-vibrations and the link between rotation and γ vibrations[12]. Gupta and Sharma compared the ARM's predictions to the transitions between the γ-g B(E2) bands in rare-earth nuclei[13]. Mittal *et al.*[14] broadened their study to include N-deficient (N>82) Tellurium-Samarium nuclides in attempt to establish a link between Z and N alterations in the B(E2) transition. They accomplished this by drawing on previous discoveries. Using the DF framework, the study[15,16] examined the continuous changes in γ with relation to N, Z. They discovered that the two variables have a high correlation. After performing study on the Grodzins equation in the context of an asymmetric rotor, they concluded that there is a clear link between it and the asymmetry variable γ. Kumari and Mittal[17] reached this result after conducting their investigation. Similarly, Bindra and Mittal[5] investigated the ESF and ROTE, which were multiplied by B(E2) values, in relation to shape-transitional isotopes. They highlighted serious flaws in their investigation and stressed the relevance of these anomalies. Over the last few years, technological improvements have enabled the collecting of a large amount of nuclear data via experimental study. As a result, theoretical calculations have been adjusted to accommodate for this volume of data. We conducted a study to investigate variations in S(4), γ0, and Grodzins product characteristics in Gd-Hf nuclei over different mass ranges. For the first time, this inquiry looks into the potential impact of the Z=64 subshell on these differences.

2 Formalism

The axial symmetry deviation in the nucleus is measured by the asymmetry parameter (γ0). The Hamiltonian describes the nucleus' energy in the triaxial rotor model. There are several strategies for computing the asymmetry parameter γ0, but the one with Rγ is recommended.[18--21]

$$\gamma_0 = 0.33\,\sin^{-1}\left[1.125\left\{1-\left(\frac{R\gamma-1}{R\gamma+1}\right)^2\right\}\right]^{1/2} \quad \text{... (1)}$$

In this case, $R\gamma = E(2^+_\gamma)/E(2^+_1)$ deviates from (∞-2). A triaxial structure can be expected for Rγ = 2, resulting in γ0 = 30°. This illustrates the relevance of γ0: the limit (Rγ ∼ ∞) with γ0 = 0° corresponds to the nucleus as an elongated ellipsoid. In the asymmetry rotor model, the lowest value for Rγ that may be obtained is 2. $E(2^+_\gamma)$ and $E(2^+_1)$ experimental values can be found on the National Nuclear Data Center (NNDC) website[22].

On the other hand for γ-bands, we will use the amount[1] to investigate the odd-even staggering:

$$S(J) = \{E(J^+_\gamma) - 2E[(J-1)^+_\gamma] + E[(J-2)^+_\gamma]\} / E(2^+_1) \quad \text{... (2)}$$

This amount, normalized to the energy of the ground band's initial excited state, 2^+_1, quantifies the displacement of the $(J-1)^+_\gamma$ level with respect to the average energy of its surrounding levels, J^+_γ and $(J-2)^+_\gamma$. S(J) is highly adaptable to changes in configuration since it takes a confound derivative form Table 1 & 2.

Table 1 — List the values of Asymmetric parameter[18] i.e. γ0 and Staggering parameter i.e. S(4) using Eq. 1,2 in the isotopes of Gd-Dy for N=88-96 Zones

Nuclei	N	Grodzin Product	S(4)	γ
^{64}Gd	88	574.95	-0.6	21.48
	90	478.75	0.03	13.86
	92	412.81	0.15	11.05
	94	399.14	0.18	10.32
	96	395.12	0.28	10.98
^{66}Dy	88	799.65	-0.59	21.98
	90	511.35	0.1	15.43
	92	460.96	0.21	12.8
	94	445.22	0.26	11.9
	96	431.53	0.28	11.96

Table 2 — List the values of Asymmetric parameter[18] i.e. γ0 and Staggering parameter i.e. S(4) using Eq. 1,2 in the isotopes of Er-Hf for N=92-102 Zones

Nuclei	N	Grodzin Product	S(4)	γ
^{68}Er	92	551	0.06	15.05
	94	511.22	0.23	13.3
	96	498.13	-0.1	12.9
	98	469.76	0.29	12.68
^{70}Yb	96	536.42	0.16	13.1
	98	489.53	0.24	11.86
	100	487.83	0.28	10.8
	102	475.61	0.32	9.27
^{72}Hf	100	425.63	0.17	11.82
	102	444.01	0.029	10.85

3. Development and Findings

3.1 For Gd-Dy isotope chains, N = 88-96

Figures 1 and 2 illustrate the S(4) values for warped Gd-Er isotope chains. Interestingly, the ratios frequently deviate from zero and usually have a strong tendency, which is the pattern observed. As the number of neutrons (N) increases, this variance becomes more noticeable. The isotope sequences of Gd-Er clearly indicate the transition from a U(5) vibrating state to an SU(3) configuration with axial symmetry resembling a rotor. At N = 92, the Gd isotope undergoes a major first-order phase change, as shown by an approximate γ0 of 11 degrees. The chains of Gd-Er isotopes have been thoroughly researched, and it has been discovered that they change from a spherical to a deformed shape. The isotopes ^{152}Gd and ^{154}Dy, in particular, are near the transition point[23-25] and display properties compatible with the X(5) model. In the nuclear chains of Er for N~100, McCutchan *et al.* [26] propose that the existence of 0_2^+ excited levels suggests the presence of a unique structure. Ahmad and Bindra[27] identified a significant difference in the energy of the initial 0^+ excited level for Er at N_B = 16 (N ~ 100) while using the Interacting Boson Model (IBA) to investigate the early collectively excited 0^+ states. Our study also revealed an unusual pattern in the S(4) of these nuclei. The constant shift in the strength of the triaxial potential in these nuclei is demonstrated by an increase in S(4) above zero with respect to the asymmetry variable in the 11-25 range. The fact that quasi-gamma band energy is at its lowest in this proximity adds to the evidence for triaxiality in N=88-90 isotones.

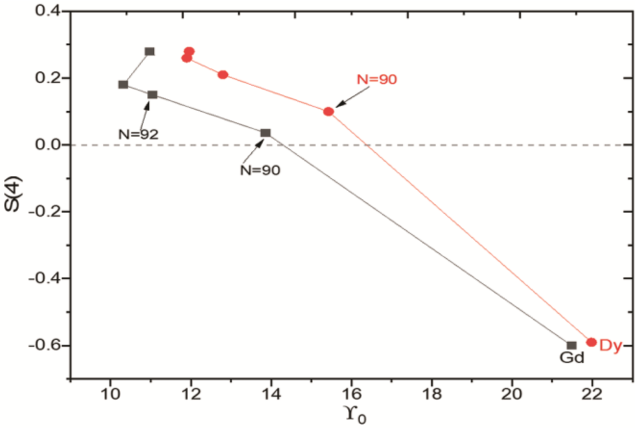

Fig. 1 — Deformed Gd-Dy isotopes exhibit unexpected fluctuations in S(4) group ratios, which intensify with increasing neutron numbers and may indicate a phase transition at N~92. Importantly, neutrons surrounding N~92 cause a divergence from zero and a shift to an SU(3) configuration. The Z=64 sub-shell also has a noticeable effect on gamma band energy in N=88–90 isotones where triaxiality is present.

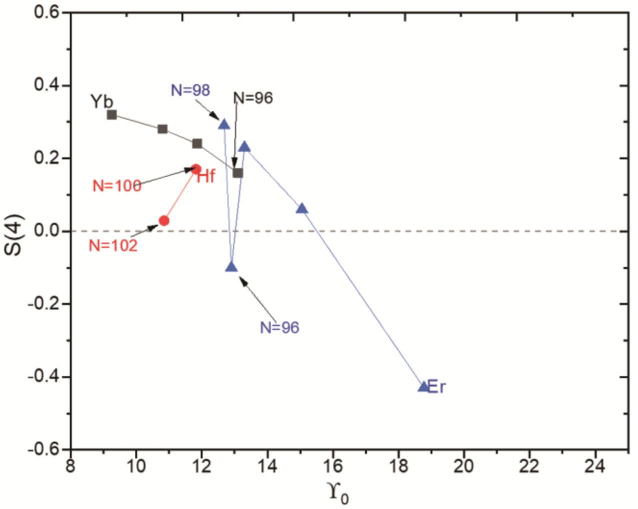

Fig. 2 — A distinctive depiction of nuclear chain patterns in Er, Yb and Hf is shown, with S(4) group values peaking at γ0 ~130 in Yb and growing exponentially at γ0 ~100 in Hf. The onset of deformation is observed around γ0~13°, a finding corroborated by Bindra and Mittal[3].

3.2 For distorted Er-Hf isotope chains, N = 92-102

In Fig. 2, we notice a distinct pattern in the nuclear chains of Errbium (Er), Ytterbium (Yb) and Hafnium (Hf). For Yb nuclei, the S(4) value tends to zero at about γ0 = 13^0, but for Hf nuclei, this value increases exponentially at about γ0 = 10^0, especially in the outermost shell. The Yb-Hf nuclear chains also showed a strong trend, as reported by Ahmad and Bindra.[27] According to their observations, the χ value for O(6) nuclei with vibrational features is almost zero but progressively increases to -1.32 in the SU(3) zone as it approaches the mid-shell (see Fig. 3 on the right in Ref. 27). This observation fits in with the

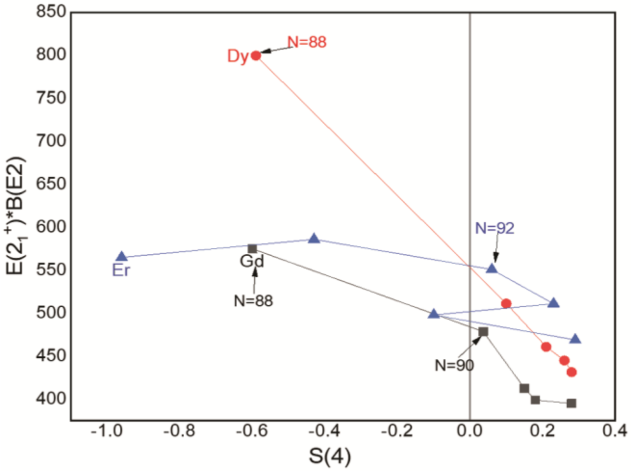

Fig. 3 — A comparative analysis of $S(4)_\gamma$, in relation with Grodzins product for Gd-Er isotopes.

distinct results we have found. When deformation begins, it is indicated by a sudden shift in the S(4) ratio, which deviates from unity at about $\gamma 0 \sim 13°$. This modification also indicates that a considerable number of nuclei have taken on a distorted conformation in this particular location (mid-shell). Furthermore, these observations are corroborated by Bindra and Mittal's findings[18]. After passing through an area with asymmetric axial forms, nuclei close to a distorted shell usually change from elongated (prolate) shapes to flattened (oblate) shapes.

Figure 3 displays the S(4) data for Gd- Er nuclei along with their Grodzins product, and the behavior is notably different. S(4) consistently enhances with $E(2_1^+)$*B(E2) reaching significant values, particularly for Gd, Er. For the heavier isotopes, $^{152, 156}$Gd, ^{154}Dy, and ^{160}Er, the S(4) and $E(2_1^+)$* B(E2) values show minimal and rather stable features. All of them, however, show a more noticeable warped pattern in the case of S(4). When it comes to isotopes with neutron numbers higher than 82 (N > 82), this distortion is noteworthy. ^{64}Gd and ^{66}Dy show abrupt kinking at N = 90, suggesting that heavy rare-earth nuclei, particularly those containing neutrons, have axially symmetric characteristics with a progressively decreasing trend beyond N > 90. According to Caikrili and Casten's results[28], this shift indicates a change from vibrator to rotor nuclei, offering strong evidence for the sudden beginning of deformation. And also, we can say that N = 90 is the point where, as shown by the Gd nuclei and the first-order phase transition after N = 92 (at N = 94), Since this study's absolute B(E2) values cover both excitation energies and relative and absolute transition probabilities, they allow for a more thorough analysis of the X(5) model predictions. Also mentioned as potential possibilities for exhibiting the traits of the X(5) critical point symmetry are ^{154}Gd and ^{156}Dy.[29]

3.3 γ-Band Energy Staggering, S(6), in relation with energy ratio, i.e., $R_{4/2}$

Figure 4 shows the changeover from U(5) (a vibrating rotor) to SU(3) (an axially symmetric rotor), which corresponds to the Gd-Hf isotope chains. According to theoretical predictions, S(6) consistently displays low values in each of these isotope chains. It transitions dynamically, starting from negative values in the more vibrational nuclei, going toward a specific isotope in the chain, and finally rising to the stiff rotor limit of 0.33. As examples, we find that $R_{4/2}$ increases noticeably and suddenly for isotopes ^{152,158}Gd, ^{154,158}Dy, and ^{156,162}Er. Specifically, we find that $R_{4/2}$ increases from 2.19 to 3.28, 2.23 to 3.20, and 2.93 to 3.23. A noteworthy change in S(6) values, from low negative values to slightly positive ones, coincides with this transition. This pattern, as seen by McCtchan *et al.*[26], is consistent with a number of nuclides, including Nd, Sm, Gd, and Dy. The data in Table 3 shows that this change usually takes place near the N∼90 isotopes in the Gd-Er range. These are recognized instances of the critical point symmetry X(5).[23-25] As a result, the passing of S(6) through or close to zero along the change from a vibrator to a distorted axially symmetric rotor may serve as a marker of a phase transition area.

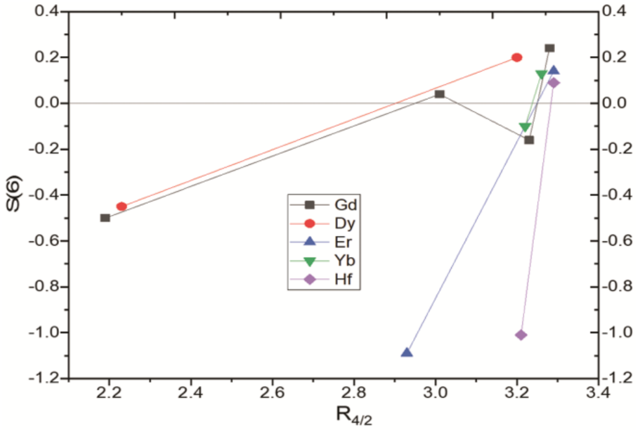

Fig. 4 — A comparative analysis of $S(6)_\gamma$, in relation with energy ratio ($R_{4/2}$) for Gd-Hf isotopes.

Table 3 — Experimental energy ratios $R_{4/2}$ and staggering S(6) [Eq. (2)] for N ~90 isotopes for Gd-Hf that are good examples of X(5) symmetry

Nuclei	S(6)	$R_{4/2}$
^{152}Gd	-0.5	2.19
^{154}Gd	0.04	3.01
^{156}Gd	-0.16	3.23
^{158}Gd	0.24	3.28
^{154}Dy	-0.45	2.23
^{158}Dy	0.20	3.20
^{156}Er	-1.09	2.93
^{162}Er	0.14	3.23
^{166}Yb	-0.10	3.22
^{168}Yb	0.13	3.26
^{176}Hf	-1.01	3.21
^{178}Hf	0.09	3.29

4 Conclusion

This work explores the concept of a "axially rigid region" where γ remains rigid and corresponds to a harmonic oscillator potential. It demonstrates how studying energy staggering along an isotopic chain can disclose information about phase transitions and changes in nuclear structure over time. The Bohr and Mottelson model, the Davydov model, and the Wilets-Jean model define the geometric properties of atomic nuclides, with γ and β as the key parameters. This research looks at excited band energy levels in Gd-Hf nuclei, focussing on how the Z=64 subshell influences these variations. The Hamiltonian describes the energy of the triaxial rotor model's nucleus, whereas the asymmetry parameter γ0 reflects its deviation from axial symmetry. The resulting $R_{4/2}$ value is the lowest. The results show that the ratios regularly vary from zero and have a dominating tendency, indicating a transition from a U(5) vibrating state to an SU(3) structure with rotor-like axial symmetry. The analysis also reveals an unusual trend in the S(4) for these nuclei, which is marked by a constant variation in the intensity of the triaxial potential. Variations in the S(4) group ratios of deformed Gd-Dy isotopes, which rise with the number of neutrons, could indicate a phase change at N~92. The Z=64 subshell has a considerable effect on the gamma band energy in the triaxial N=88-90 isotones. The S(4) value in nuclear chains of Er, Yb, and Hf tends to zero about γ0 = 130. For Hf nuclei, this value increases exponentially at around γ0 = 100, especially in the outermost shell. Strong trending was also observed in the Yb-Hf nuclear chains, where a sudden change in the S(4) ratio indicated a disrupted conformation.

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