

Selection of raw material parameters for multi-response optimization of cotton yarn qualities

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In this work, a multi-response optimization of cotton yarn quality using desirability function approach has been attempted. Being a natural product, cotton yarn qualities are primarily governed by raw material characteristics. This work aims to resolve the complexity of simultaneous optimization of raw material properties using a hybrid multi-response optimization model, where predictive power of support vector regression and optimization capability of genetic algorithm are employed with desirability function. The individual desirability of cotton fibre qualities is assessed from the six properties, such as fibre strength, elongation, fineness, upper half mean length, uniformity index and short fibre content. The yarn quality parameters, such as yarn strength, yarn elongation, hairiness and unevenness, are combined together to express overall desirability. The optimum cotton quality parameters essential to produce good quality yarn can be determined from the proposed multi-response optimization model.

Keywords: Cotton fibre, Desirability function, Fibre properties, Genetic algorithm, Support vector regression, Yarn quality

1 Introduction

Appropriate selection of raw materials is the most difficult task for all yarn manufacturers. It is a cherished goal for every spinning industry to pick the raw materials with optimum properties. The cotton yarn quality is largely governed by the fibre properties. The relationship between the fibre and yarn properties is highly non-linear. Large number of publications has been reported in the literature on the prediction of yarn quality based on the raw material data¹⁻⁸. Bulk of such publications focuses on the prediction of yarn strength from the fibre properties. However, very limited information is available to date on yarn engineering which is basically a reverse approach to the prediction of yarn quality from the properties of raw materials⁹. Das *et al.*¹⁰ developed a hybrid artificial neural network-genetic algorithm (ANN- GA) approach to select optimum fibre properties to produce an engineered yarn with predefined yarn strength. In the ANN-GA model, only one yarn property can be optimized at a time. The idea of this work is to propose a model to mitigate the complexity of the simultaneous optimization of yarn properties, such as strength, elongation, hairiness and unevenness.

In last few years, ANN has opened a new horizon in the prediction modeling of yarn properties, which generally surpasses the mathematical and statistical models in terms of prediction accuracy. In recent years, Support Vector Regression (SVR) is gaining further attentions because of the better generalization capability and higher predictive power as compared to ANN¹¹. Hence, SVR model is attempted in this work to develop the equations for prediction of cotton yarn strength, yarn elongation, hairiness and unevenness from the given fibre properties. A search based optimization technique like GA deriving from the concept of natural selection and survival of the fittest uses the SVR model to search the optimum fibres properties in order to spin a yarn with specific quality parameters for a given spinning system.

This paper uses desirability function approach to determine the optimum parameters of cotton fibre properties, such as fibre strength (FS), fibre elongation (FE), upper half mean length (UHML), length uniformity index (UI), fineness ($\mu\text{g}/\text{inch}$) and short fibre content (SFC) for simultaneous optimization of yarn strength, yarn elongation, hairiness and unevenness (U%).

2 Methodology

2.1 Concept of SVR

Support vector machine (SVM) is a learning system that uses a hypothesis space of linear functions

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in a high dimensional feature space, trained with a learning algorithm from optimization theory that implements a learning bias derived from statistical learning theory¹². SVM can be applied for classification as well as regression or functional approximation problems. In Support Vector Regression (SVR), the experimental dataset is mapped into a high-dimensional feature space. Considering a training dataset $S = \{[x_i, y_i] \in \mathbb{R}^n \times \mathbb{R}, i = 1, \dots, l\}$ consisting of l pairs of data points, the aim of SVR is to obtain a function f which best approximates the system responses from S in the following form:

$$y = f(\mathbf{x}) = \langle \mathbf{w} \cdot \varphi(\mathbf{x}) \rangle + b \quad \dots (1)$$

where the inputs $x, \in \text{and } \mathbb{R}^n$ are the n -dimensional vectors and system responses; $y, \in \text{and } \mathbb{R}$, continuous values; $\varphi(\mathbf{x})$, the mapping function which maps from \mathbb{R}^n to higher dimensional feature space F ; \mathbf{w} , the weight vector; and b , the bias term. SVR models are equipped with high generalization accuracy due to introduction of Vapnik's ε -insensitive loss function¹³, which has the following form:

$$L^\varepsilon(\mathbf{x}, y, f) = |y - f(\mathbf{x})|_\varepsilon = \max(0, |y - f(\mathbf{x})| - \varepsilon) \quad \dots (2)$$

where f is a real-valued function on the input domain X ; and $\mathbf{x} \in X, y \in \mathbb{R}$ and ε , a precision parameter representing the radius of the tube located around the regression function^{12,13}. For optimizing the generalization performance of the regression, the primal problem can be defined as follows:

$$\begin{aligned} \text{Minimize} \quad & \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^l (\xi_i + \hat{\xi}_i) \\ \text{subject to} \quad & (\langle \mathbf{w} \cdot \varphi(\mathbf{x}_i) \rangle + b) - y_i \leq \varepsilon + \xi_i, \quad \dots (3) \\ & y_i - (\langle \mathbf{w} \cdot \varphi(\mathbf{x}_i) \rangle + b) \leq \varepsilon + \hat{\xi}_i, \\ & \xi_i, \hat{\xi}_i \geq 0, \quad i = 1, 2, \dots, l. \end{aligned}$$

where C is a pre-specified value; and $\xi, \hat{\xi}$ are slack variables representing upper and lower constraints on the outputs of the system. The corresponding Lagrangian function for this optimization problem is

given by the following equation with $\alpha_i \geq 0, \hat{\alpha}_i \geq 0, \beta_i \geq 0$ and $\hat{\beta}_i \geq 0$:

$$\begin{aligned} L(\mathbf{w}, b, \xi, \hat{\xi}, \alpha, \hat{\alpha}, \beta, \hat{\beta}) = & \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^l (\xi_i + \hat{\xi}_i) \\ & + \sum_{i=1}^l \alpha_i \left[\left(\sum_{i=1}^l \langle \mathbf{w} \cdot \varphi(\mathbf{x}_i) \rangle + b \right) - y_i - \varepsilon - \xi_i \right] \\ & + \sum_{i=1}^l \hat{\alpha}_i \left[y_i - \left(\sum_{i=1}^l \langle \mathbf{w} \cdot \varphi(\mathbf{x}_i) \rangle + b \right) - \varepsilon - \hat{\xi}_i \right] - \sum_{i=1}^l \beta_i \xi_i - \sum_{i=1}^l \hat{\beta}_i \hat{\xi}_i \end{aligned} \quad \dots (4)$$

The coefficients $\alpha_i, \hat{\alpha}_i, \beta_i$ and $\hat{\beta}_i$ are called the Lagrange multipliers, that result in nonzero values after the optimization, provided they satisfy the Karush-Kuhn-Tucker (KKT) conditions. The corresponding dual is found by differentiating the Lagrangian function of Eq. (4) with respect to $\mathbf{w}, b, \xi, \hat{\xi}$ and re-substituting the relations obtained into the primal with some simplification. The dual problem is given by

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^l (\hat{\alpha}_i - \alpha_i) y_i - \varepsilon \sum_{i=1}^l (\hat{\alpha}_i + \alpha_i) \\ & - \frac{1}{2} \sum_{i,j=1}^l (\hat{\alpha}_i - \alpha_i) (\hat{\alpha}_j - \alpha_j) \langle \phi(\mathbf{x}_i) \cdot \phi(\mathbf{x}_j) \rangle \\ \text{subject to} \quad & 0 \leq \alpha_i, \hat{\alpha}_i \leq C, \quad i = 1, \dots, l \\ & \sum_{i=1}^l (\hat{\alpha}_i - \alpha_i) = 0, \quad i = 1, \dots, l \end{aligned} \quad \dots (5)$$

The corresponding KKT complementarities conditions are

$$\begin{aligned} \alpha_i (\langle \mathbf{w} \cdot \varphi(\mathbf{x}_i) \rangle + b - y_i - \varepsilon - \xi_i) &= 0, \\ \hat{\alpha}_i (y_i - \langle \mathbf{w} \cdot \varphi(\mathbf{x}_i) \rangle - b - \varepsilon - \hat{\xi}_i) &= 0, \quad \dots (6) \\ \xi_i \hat{\xi}_i &= 0, \quad \alpha_i \hat{\alpha}_i = 0, \\ (\alpha - C) \xi &= 0, \quad (\hat{\alpha} - C) \hat{\xi} = 0 \end{aligned}$$

where $i = 1, \dots, l$. The solution of the dual problem in Eq. (5) is given by

$$f(\mathbf{x}) = \sum_{i=1}^l (\hat{\alpha}_i - \alpha_i) K(\mathbf{x}_i, \mathbf{x}) + \bar{b} \quad \dots (7)$$

where \bar{b} is chosen so that $f(\mathbf{x}_i) - y_i = -\varepsilon$ for any i with $0 < \bar{\alpha}_i, \bar{\alpha}_i < C$. Using the relation $\hat{\alpha}_i \alpha_i = 0$ of the KKT conditions, the support vectors (SVs) are points where exactly one of the Lagrange multipliers is greater than zero. Therefore, if we consider the band of $\pm \varepsilon$ around the function output by the learning algorithm, the points that are not strictly inside the tube are SVs and those not touching the tube will have the absolute value of that parameter equal to C . For data points inside the tube, both Lagrange multipliers will be equal to zero. The function $K(\mathbf{x}_i, \mathbf{x})$

in Eq. (7) is a kernel function, which projects the data into a high dimensional feature space and thereby increases the computational power of the linear learning machine, has the following form:

$$K(x, z) = \langle \varphi(x) \cdot \varphi(z) \rangle \quad \dots (8)$$

where φ is a mapping from input space X to an inner product feature space F for all $x, z \in X$. The use of kernels makes it possible to map the data implicitly into a feature space and to train a linear machine in such a space, potentially side-stepping the computational problems inherent in evaluating the feature map.

2.2 Concept of GA

Genetic algorithm (GA) is an unorthodox search method based on the natural selection process for solving complicated optimization problems. Holland of the University of Michigan developed it in the early 1970s¹⁴. Unlike traditional optimization algorithms, GA is a global and robust optimization method, based on the evolutionary behavior of biological life forms in nature. One of the outstanding differences between GA and classical optimization algorithms is that the latter use a point-by-point approach, where one solution in each iteration is modified to a different (hopefully better) solution and eventually the outcome becomes a local optimized solution. In GA, a population of solutions works in each iteration instead of a single solution. As multiple solutions are processed simultaneously, it is very likely that the expected optimized GA solution should turn out to be a global one. In the context of design, the merits of GA have been numerous and apply in various fields.

GA is a heuristic search algorithm that can be applied when the dimension of the data space is too large for an exhaustive search. GA proceeds first by

randomly generating an initial population of individuals, which should ideally cover the domain to explore. Each individual is represented by a binary coded string or chromosome encoding a possible solution in the data space. At every iteration step or generation, the individuals in the current population are selected for mating. The selection of parents for mating is based on their fitness for a particular objective function. A roulette wheel selection mechanism is the most preferred method for selection. Selection alone cannot introduce new individuals into the population, which is necessary in order to make the solution as independent of the initial population as possible. New individuals in the search space are thus generated by two operations, viz. crossover and mutation. Crossover is the main genetic operator. It operates on two selected individuals (parents) at a time and generates two new individuals (offsprings) by combining both chromosomes features. The crossover operation is not always applied to all selected chromosomes. The application of crossover is governed by a crossover probability, denoted by p_c . If p_c was set 0.8 for the crossover, meaning that an individual has 80 % of chance for undergoing crossover. The mutation operation is used as a means to inject new features, which are absent in both the parent strings. Thus, if a solution gets stuck at the local minimum, mutation may help it to come out of this situation and consequently, it may jump to global basin. The mutation consists in flipping bits of individual's strings at random from 0 to 1 or vice versa. The typical range for p_c is 0.6-0.8, while the range of mutation probability (p_m) is generally chosen between 0.01 and 0.001¹⁵. Higher mutation probability is not desirable, as it may spoil the good individuals.

2.3 Concept of Desirability Function

The desirability function approach to simultaneously optimizing multiple equations was originally proposed by Harrington¹⁶. Derringer and Suich¹⁷ popularized the approach by introducing more general transformation of the response into desirability function. Desirability function is one of the most popular multi-response optimization technique used in industry. In desirability function method, the product is considered as completely unacceptable if any one quality out of multiple quality characteristics of the product falls outside the desired limit. Desirability function is defined individually for each response parameters with goals and boundaries.

The individual desirability (d_i) has three types of goals, namely maximize the response, minimize the response and target the response which are expressed in following equations, respectively.

$$d_i = \begin{cases} 0 & \text{if } Y_i \leq Y_{min} \\ \left(\frac{Y_i - Y_{min}}{Y_t - Y_{min}}\right)^p & \text{if } Y_{min} \leq Y_i \leq Y_t \\ 1 & \text{if } Y_i \geq Y_t \end{cases} \quad \dots (9)$$

$$d_i = \begin{cases} 1 & \text{if } Y_i \leq Y_t \\ \left(\frac{Y_i - Y_{max}}{Y_t - Y_{max}}\right)^q & \text{if } Y_t \leq Y_i \leq Y_{max} \\ 0 & \text{if } Y_i \geq Y_{max} \end{cases} \quad \dots (10)$$

$$d_i = \begin{cases} 0 & \text{if } Y_i \leq Y_{min} \\ \left(\frac{Y_i - Y_{min}}{Y_t - Y_{min}}\right)^p & \text{if } Y_{min} \leq Y_i \leq Y_t \\ \left(\frac{Y_i - Y_{max}}{Y_t - Y_{max}}\right)^q & \text{if } Y_t \leq Y_i \leq Y_{max} \\ 0 & \text{if } Y_i \geq Y_{max} \end{cases} \quad \dots (11)$$

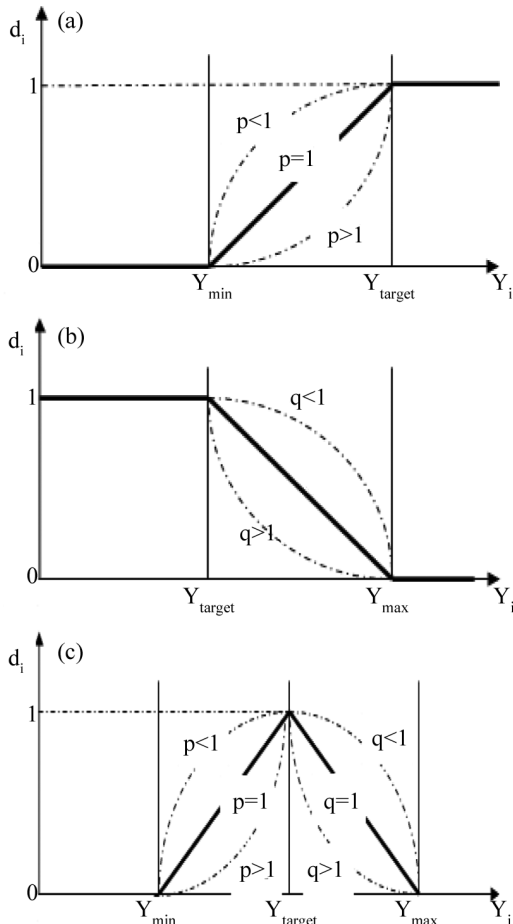


Fig. 1 — Desirability function to (a) maximize, (b) minimize and (c) reach a target value

The desirability functions with different goals and boundaries are shown in Fig. 1. The individual desirability (d_i) assigns a value between 0 and 1 to the each response (Y_i), and $d_i = 0$ represents the completely undesirable value of the response (Y_i) and $d_i = 1$ represents the ideal response value. The exponents' p and q in the Eqs (9)-(11) determine the degree of importance to hit the target value. The desirability function approaches linearly for $p = q = 1$. The desirability function is convex for $p < 1$, $q < 1$ and is concave for $p > 1$, $q > 1$. The d_i values are combined to calculate the 'overall desirability function' for optimization. The equation of 'overall desirability function' is manifested in the following equation:

$$D = \sqrt[w]{(d_1^{w_1} \times d_2^{w_2} \times \dots \times d_n^{w_n})} \quad \dots (12)$$

where $w = \sum w_i$; w_i , the weight of i^{th} response; and n , the number of responses. Both the 'individual desirability function' and 'overall desirability function' have a range from 0 to 1. If any individual desirability (d_i) of the response is completely undesirable then overall desirability (D) becomes 0.

2.4 Hybrid SVR-GA-Desirability Function Model

The data of 40 types of cotton fibres and corresponding carded yarns of 20's Ne nominal count made from each fibre type in ring spinning technology are collected from the industry. The fibre data encompasses the properties, such as strength (FS), elongation (FE), upper half mean length (UHML), length uniformity index (UI), fineness ($\mu\text{g}/\text{inch}$) and short fibre content (SFC), are used as inputs to the SVR model. The corresponding yarn properties, such as yarn strength, yarn elongation, hairiness and unevenness (U%), are considered as output in the model. The

Table 1 — Summary of fibre and yarn properties

Parameter	Minimum	Maximum	Mean	Standard deviation
Fibre parameters				
FS, cN/tex	26.50	34.0	29.02	1.530
FE, %	5.30	6.90	6.28	0.462
UHML, inch	0.96	1.20	1.05	0.048
UI	79.10	83.20	81.41	1.065
FF, $\mu\text{g}/\text{in}$	3.10	5.00	4.21	0.447
SFC, %	5.60	18.40	9.67	2.862
Yarn parameters				
Tenacity, cN/tex	13.10	18.00	14.96	1.108
Elongation, %	4.23	6.89	5.68	0.712
Unevenness, U%	10.90	15.80	12.93	1.196
Hairiness index	4.64	6.08	5.31	0.348

summary of the dataset are presented in Table 1. The dataset was used to train the SVR model.

The randomly selected 32 datasets are used for training and remaining 8 datasets are used for testing the SVR model. The trained SVR model is then used to predict the yarn strength, yarn elongation, hairiness and unevenness by optimizing the overall desirability. GA is used for optimization of fibre properties using

multi response desirability function for obtaining the target values of cotton yarn properties. The minimum and maximum values of yarn strength, yarn elongation, hairiness and unevenness are estimated from the dataset and the target values were set based on the experience of commercial spinners for 20's Ne carded cotton yarns which are shown in Table 1. In this study, the values of p and q were chosen as 1. The

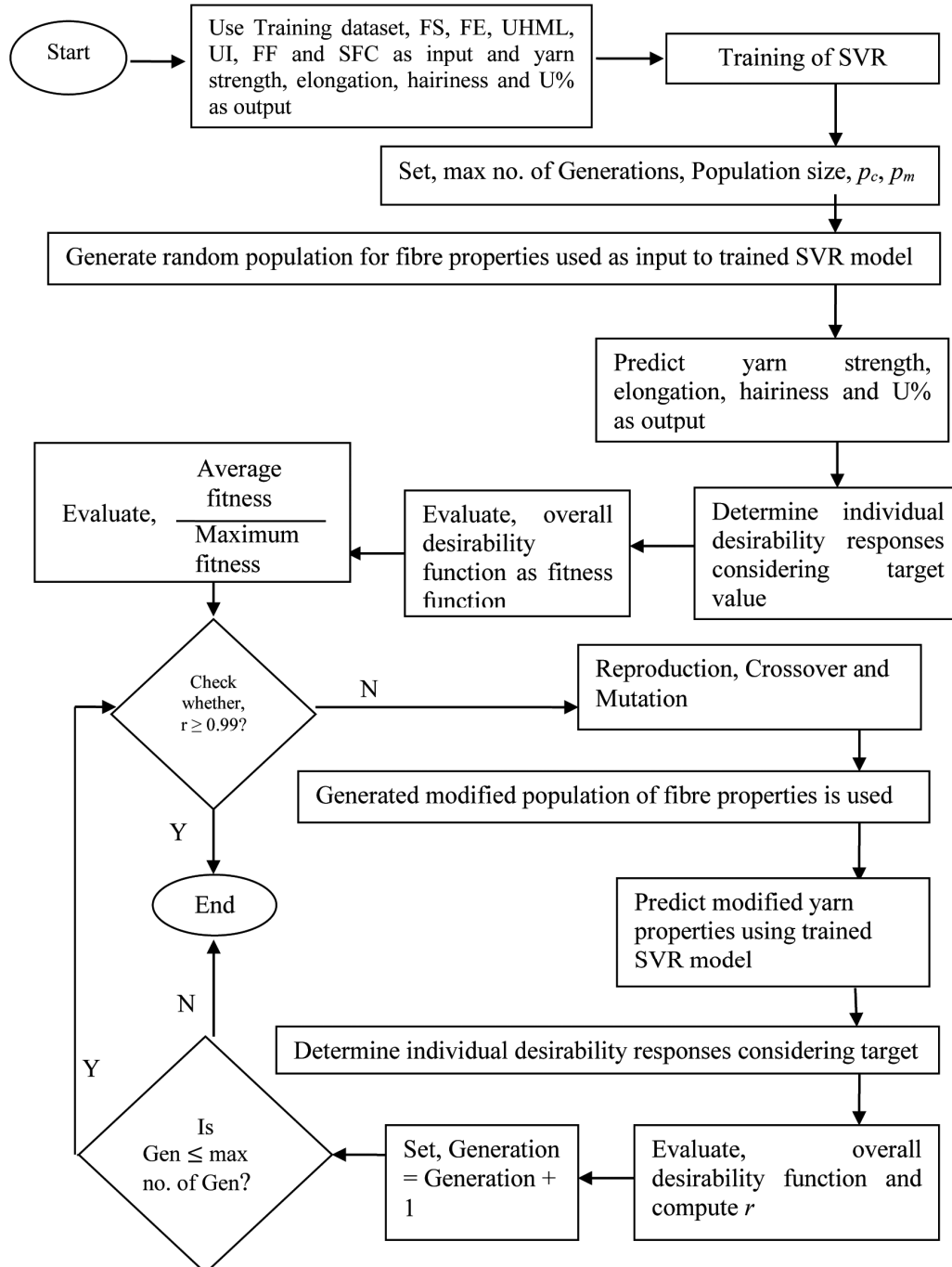


Fig. 2 — Flowchart of hybrid SVR-GA and desirability function model.

diagrams of the ‘individual desirability’ of yarn properties are depicted in Fig 2. The optimization problem of maximizing the ‘overall desirability’ was solved for using MATLAB optimization toolbox (version 7.7). The weight value (w_i) was chosen equal to 1 which implies equal importance among responses.

Initially, a population size of 2000, each individual of which consists of a set of six randomly generated elements representing the fibre properties within the specified bounds is generated. Each individual of the population is used as inputs to the trained SVR model for obtaining the predicted output of yarn properties. The desirability function in Eq. (11) is evaluated for whole population from which the ratio of the average fitness value to the maximum fitness value (r) is calculated. The population of input elements is then modified using different operators of GA, namely reproduction, cross-over and mutation. A new modified population of fibre properties is now formed which are again used as input of trained SVR model to obtain the predicted yarn properties. The fitness is re-evaluated and r is recalculated. This completes the first generation of GA. The GA runs for generation after generation until it satisfies the termination criteria, which is either r attains a desired value or number of generations reaches to a maximum value. The desired value of r is chosen as 0.99, meaning 99% of the population converges to the optimum fitness value. Maximum number of generation is set to 1000. The optimum solution of GA is obtained with the values of 0.7 and 0.001 for p_c and p_m respectively. Roulette wheel selection scheme is applied for reproduction operation to select the good individuals of weights from the population on the basis of their fitness information. Uniform cross-over method is applied to form new individual of fibre data. A flowchart of the hybrid SVR-GA model to optimize the fibre properties in cotton selection problem using desirability function is illustrated in Fig. 2.

3 Results and Discussion

The training mean error percentage and testing mean error percentage of SVR model are represented in Table 2. It can be inferred that the testing error percentage is expectedly higher than the training accuracy because the former is done on the unknown data. This demonstrates the ability of the SVR model to respond reasonably good to unknown inputs as well. The trained SVR model is used to predict the yarn properties which are then used to generate individual desirability function considering

Table 2 — Prediction Performance of SVR

Yarn properties	Training mean error %	Testing mean error %
Strength, cN/tex	2.33	3.15
Elongation, %	4.07	4.37
Unevenness, U%	2.54	6.37
Hairiness Index	1.94	4.33

Table 3 — Obtained yarn property and individual desirability

Yarn properties	Target	Obtained from the model	Individual desirability value
Strength, cN/tex	16.50	16.17	0.9034
Elongation, %	6.00	5.90	0.9478
Unevenness, U%	11.00	11.51	0.8922
Hairiness index	4.80	4.83	0.9754

respective target values. ‘Maximize the response’ function is followed to depict individual desirability for yarn strength and elongation using Eq. (9). However, individual desirability for yarn unevenness and hairiness are obtained following ‘minimize the response’ function as shown in Eq. (10). Table 3 demonstrates the target values and predicted values of yarn properties along with their desirabilities. The individual desirability values are 0.9034, 0.9478, 0.8922 and 0.9754 for yarn strength, elongation, unevenness and hairiness respectively. The overall desirability is depicted as 0.9291 for this hybrid optimization model. Figure 3 shows the desirability functions for yarn strength, elongation, unevenness and hairiness.

The optimized result as 31.51, 6.83, 1.00, 82.84, 4.02 and 5.63 for FS, FE, UHML, UI, FF and SFC respectively, is obtained using desirability function for the multi response optimization by hybrid SVR-GA model. These are the optimum combination of fibre properties that can engender a yarn with predefined properties. The model is able to maximize the overall desirability function such that target values can be achieved. The proposed hybrid model is a scientific approach which can be adopted for other textile materials as well.

For the purpose of validation, the optimized parameters are compared with a cotton lot having the closest values with the obtained parameters. The observed cotton lot with the parameters as 30.9, 6.9, 1.01, 82.6, 3.7 and 6.2 for FS, FE, UHML, UI, FF and SFC respectively, is obtained from the dataset. The yarn properties such as strength, elongation, unevenness and hairiness, for the lot are found to be 16.2, 5.96, 11.9 and 4.98 respectively, which corresponds to the individual desirability of 0.9118, 0.9774, 0.8125 and 0.8594 respectively. The overall desirability for the lot is

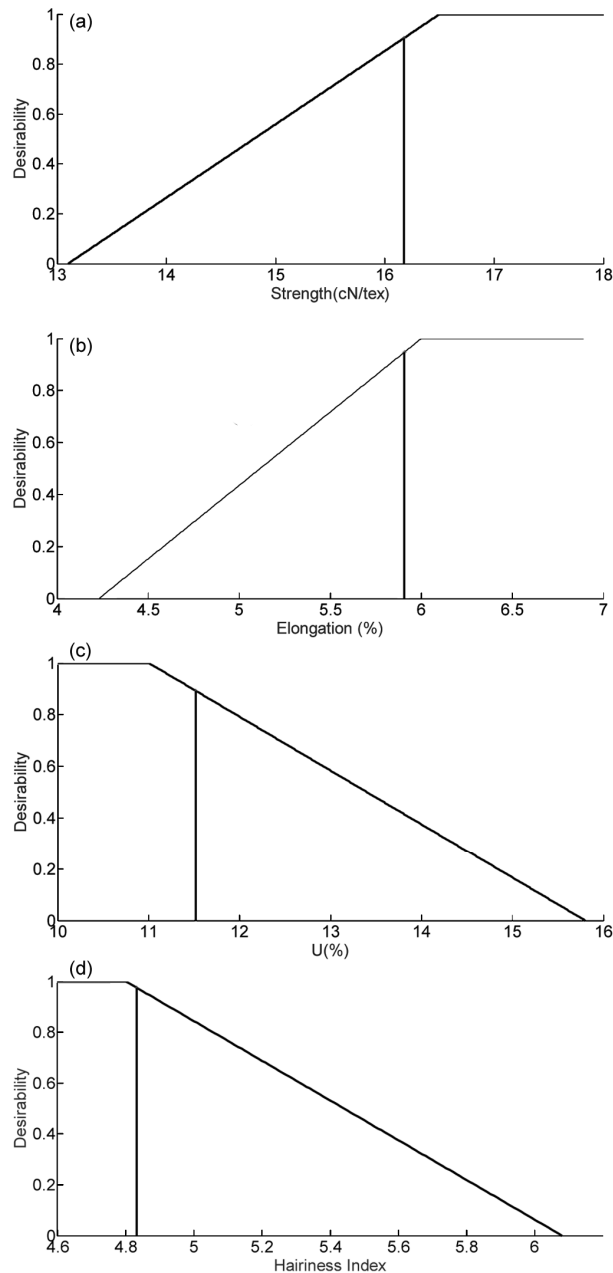


Fig. 3 — Individual desirability for (a) yarn strength, (b) yarn elongation, (c) unevenness and (d) hairiness index

estimated to be 0.8882. Table 4 represents the values of observed yarn properties from the dataset and corresponding desirability. The comparison of the observed and optimized values of the output parameters for the cotton lot shows a satisfactory agreement.

4 Conclusion

A hybrid model has been developed combining SVR and GA for multi response optimization with

Table 4 — Validation with actual data

Yarn properties	Actual	Desirability value
Strength, cN/tex	16.20	0.9118
Elongation, %	5.96	0.9774
Unevenness, U%	11.90	0.8125
Hairiness index	4.98	0.8594

desirability function for selection of raw material quality for cotton spinning industry. The proposed hybrid model captures both the high prediction power of SVR and global solution searching ability of GA. The cotton fibre properties, such as FS, FE, UHML, UI, FF and SFC, have been optimized using the proposed model. The overall desirability index is maximized using SVR-GA model for optimization of cotton properties against the target yarn quality values. The proposed model obtained a set of values for FS, FE, UHML, UI, FF, and SFC to engineer the yarn with desired combination of yarn strength, elongation, unevenness and hairiness. The overall desirability of 0.9291 has been achieved, resulting from individual desirability of are 0.9034, 0.9478, 0.8922 and 0.9754 for strength, elongation, unevenness and hairiness respectively.

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