

## Variable thickness with Ohmic heating and viscous dissipation effect on MHD Casson-nanofluid flow through a porous media

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The magneto-hydrodynamic (MHD) flow of non-Newtonian nanofluid with variable thermal conductivity past a moving surface of variable thickness has been investigated in the present work. The flow in this discussion obeys Casson model through a porous medium. Moreover, the effects of thermal radiation, heat generation, Ohmic heating, viscous dissipation and chemical reaction are taken into account. The governing non-linear partial differential equations (PDEs) which describe the velocity, temperature and nanoparticle concentration are converted to a non-linear system of ordinary differential equations (ODEs) using similarities transformation. The obtained system of equations is solved by using a numerical technique with the help of shooting method. The impacts of various parameters on the fluid behaviour are discussed and illustrated graphically via a set of figures. It is found that the velocity increases with increasing of the magnetic field parameter. Also, the temperature increases as the thermal radiation parameter rises. Moreover, an increment in the Brownian motion parameter causes a reduction in nanoparticle concentration.

**Keywords:** Non-Newtonian fluid, Ohmic effect, Porous media, Viscous dissipation

### Introduction

Over the past decades, the study of convective heat transfer in nanofluids is in the limelight. Fluids such as water, oil & ethylene glycol used in various industrial processes have poor thermal conductivity. In opposite, solid materials have remarkable thermal conductivity when compared to fluids. Choi<sup>1</sup> first initiated the idea of nanofluids to overcome the problem of poor thermal conductivity of convective fluids. He synthesised nanofluids by suspending nanometer-sized particles in regular fluids and observed an enhancement in heat transfer ability of resultant fluid (nanofluid). carbides (such as SiC), nitrides (same as AlN, SiN), metals (for example Al, Cu), Metal oxides (same as Al<sub>2</sub>O<sub>3</sub>, TiO<sub>2</sub>), or nanotubes with diameter ranges from 10 to 100 nm are excellent examples of nanoparticles used in synthesis of nanofluids. Nanofluids have gained vast interest among researchers due to the enhancement of their thermo-physical properties such as thermal conductivity, thermal diffusivity and convective heat transfer coefficients. Noreen *et al.*<sup>2</sup> discussed the influence of copper particles in the flow of blood in a vertical channel. The problem of the flow of a biviscosity nanofluid between two rotating disks

through a porous medium with Cattaneo-Christov heat flux is analyzed by Abou-zeid<sup>3</sup>. Farooq *et al.*<sup>4</sup> have examined the flow of Carreau nanofluid containing gyrotactic microorganisms over a stretched cylinder. Abuiyada *et al.*<sup>5</sup> used HPM to solve peristaltic flow of Bingham plastic nanofluid through a vertical symmetric channel under the effects of Joule heating, radiation and chemical reaction. Eldabe *et al.*<sup>6</sup> have studied the flow of non-Newtonian nanofluid (Al<sub>2</sub>O<sub>3</sub>) through the boundary-layer containing gyrotactic microorganisms. Bhatti *et al.*<sup>7</sup> have investigated the flow of a hybrid nanofluid through an elastic circular surface having a non-Darcy porous medium. The base fluid was supposed to be water, which is electrically conducting in the presence of an extrinsic magnetic field. The base fluid contained cobalt oxide and grapheme nanoparticles (Co<sub>3</sub>O<sub>4</sub>-Go/NPs). There are many authors who have presented research related to nanofluids<sup>8-14</sup>.

The study of magneto-hydrodynamic (MHD) is crucial in fluid mechanics particularly if the fluid under investigation is non-Newtonian. MHD plays an important role in numerous applications such as cooling of nuclear reactors, power generation system, plasma jets, small-vessel blood flow and cancer

therapy. Moreover, MHD is of importance in astrophysical and geophysical sciences, agricultural engineering, chemical synthesis and petroleum industries. Thus, there are many studies that investigate MHD flows. The boundary layer flow of the MHD Sisko fluid model over the stretched cylinder and temperature with the influence of viscous dissipation was studied by Malik *et al.*<sup>15</sup>. MHD boundary layer flow of Jeffrey nanofluid by a nonlinear stretching surface with heat generation/absorption was investigated by Hayat *et al.*<sup>16</sup>. Ramesh and Devkar<sup>17</sup> examined the influence of heat transfer on the peristaltic transport of an incompressible MHD second grade fluid through a porous medium in an inclined asymmetric channel. Reddy *et al.*<sup>18</sup> studied the magneto and chemical reaction effects involving H<sub>2</sub>O-based Ag and Cu nanoparticles over a rotating disk saturated by porous medium. Eldabe *et al.*<sup>19</sup> analyzed the MHD peristaltic flow of a pseudoplastic nanofluid with variable viscosity through a porous medium in the presence of Ohmic dissipation and chemical reaction. Eldabe *et al.*<sup>20</sup> studied the effects of internal heat generation and mixed convection with thermal radiation on peristaltic motion of a non-Newtonian fluid which obeys third-grade model. The flow is under the effect of radially varying magnetic field. Ibrahim and Abouzeid<sup>21</sup> discussed the impacts of variable velocity slip and activation energy on MHD peristaltic flow of Prandtl nanofluid through a non-uniform channel. The literature on the line of MHD flow and their relation to heat and mass transfer is in literatures<sup>22-32</sup>.

Non-Newtonian fluids have obtained the concern of researchers based on their superior characteristics. Such fluids are used in quite a few natural and industrial applications such as molten polymers, volcanic lava, drilling mud, polycrystal melt, oils, some paints, fluid suspensions, food products, cosmetics and many others. Moreover, most of the physiological fluids cannot be described by the Newtonian model. Therefore, several non-Newtonian models are developed to investigate the flow behaviour of these fluids. The study of non-Newtonian fluids and its applications is the subject of a significant amount of literature. The flow of an unsteady Powell-Eyring nanofluid across a shrinking sheet with effects of heat generation and thermal radiation was studied by Agbaje *et al.*<sup>33</sup>. The effects of Joule heating and viscous dissipation on Casson nanofluid flow across a stretching sheet of variable

thickness were investigated by Reddy *et al.*<sup>34</sup>. Eldabe *et al.*<sup>35</sup> discussed the effects of thermal diffusion and diffusion thermo on the transport of a non-Newtonian Eyring Powell nanofluid containing gyrotactic microorganisms through the boundary layer. The intra-uterine flow with small suspended particles under the influence of heat transfer was investigated by Bhatti *et al.*<sup>36</sup>. Eldabe *et al.*<sup>37</sup> have studied the peristaltic flow of MHD non-Newtonian Bingham nanofluid in a uniform symmetric channel with wall properties. Eldabe *et al.*<sup>38</sup> studied the MHD peristaltic flow of a third-grade nanofluid with heat transfer between two co-axial tubes is done. The effects of porous medium, magnetic field, heat generation, and chemical reaction are taken into consideration. Following it, a number of researchers used modeling of non-Newtonian fluids to get the solutions of some engineering problems with various conditions<sup>39-50</sup>.

Researchers have focused much interest on the study of boundary layer flow over a stretching sheet due to its various applications in engineering and industrial processes, for instance in the production and extraction of rubber and polymer sheets. Accordingly, several researchers have investigated different aspects of stretching sheet problems. Abou-Zeid<sup>51</sup> discussed the impact of heat transfer on MHD boundary-layer transport across a non-isothermal stretching sheet. The impact of thermal radiation with, porosity parameter, heat source/sink and slip on Cu-water nanofluid flow across a stretching cylinder was investigated by Pandey and Kumar<sup>52</sup>. Ismail *et al.*<sup>53</sup> analyzed the effect of variable thermal conductivity on the boundary layer of Casson nanofluid flow over a moving surface with a non-uniform thickness. Eldabe *et al.*<sup>54</sup> studied the boundary-layer motion of Casson fluid with heat and mass transfer through porous medium past a shrinking surface. Mahanthesh *et al.*<sup>55</sup> studied the unsteady MHD free convection flow of a fluid past a vertical moving/stationary plate in the presence of nanoparticles, thermal radiation, heat source/sink and chemical reaction effects.

The present work is an extension to Ismail *et al.*<sup>53</sup> to include the effects of viscous dissipation with Ohmic heating through a porous medium. Moreover, chemical reaction is taken into consideration. Therefore, we are concerned with studying the MHD boundary-layer flow of Casson nanofluid over a moving surface under the effects of thermal radiation, heat source, Ohmic heating, viscous dissipation and chemical reaction. The governing PDEs are converted

to ODEs by using the similarity transformation. Our system is numerically solved under the appropriate boundary conditions in order to obtain the velocity, temperature, and nanoparticle concentration distributions. The influences of the physical parameters of the problem on these solutions are discussed and illustrated graphically through a number of figures.

**Formulation of the problem**

Let us consider the two-dimensional steady flow with variable thermal conductivity  $k$  of an incompressible Casson nanofluid through a porous medium over a moving surface of variable thickness. We use the Cartesian coordinates system  $(x, y)$  such that the surface is subjected to move with velocity  $U_{w(x)} = a(x + b)^n$  along the  $x$ -axis direction and  $y$ -axis is normal to it. Here  $a, b$  are positive constants and  $n$  is the shape parameter. Moreover, the surface is described with a given profile which is specified as  $y = \delta(x + b)^{\frac{(1-n)}{2}}$ . Here the  $\delta$  coefficient is small to describe the flow along the thin surface and to avoid the pressure gradient near the plate. We assumed  $n \neq 1$  for the entire problem as  $n = 1$  corresponds to the flow via flat surface.

The rheological equation of state for an isotropic and incompressible flow of Casson fluid is given by:

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi \succ \pi_c \\ 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi \prec \pi_c \end{cases}$$

In the above equation  $\mu_B$  is the plastic dynamic viscosity of the non-Newtonian fluid,  $p_y$  is the yield stress of the fluid,  $e_{ij}$  is the  $(i, j)$ th component of the deformation rate,  $\pi = e_{ij} e_{ij}$  and  $\pi_c$  the critical value of  $\pi$  based on the non-Newtonian model.

The boundary layer is affected by heat source  $Q(x)$ , thermal radiation with heat flux  $q_r$  and varying magnetic field  $B(x)$  which is exerted perpendicularly to the surface. Effects of endothermic chemical reaction<sup>56</sup>, Ohmic heating and viscous dissipation are taken into consideration.

It is assumed that the fluid temperature  $T(x, y)$  and the nanoparticles concentration  $C(x, y)$  take constant values at the moving surface  $T_w$  and  $C_w$ , respectively, while their ambient values are denoted respectively by  $T_\infty$  and  $C_\infty$ .

The boundary layer equations governing the flow in view of the above conditions are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B^2(x)}{\rho} + \frac{\nu}{k_0}\right) u \quad \dots (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y}\right) + \tau \left[ D_B \left(\frac{\partial C}{\partial y} \frac{\partial T}{\partial y}\right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y}\right)^2 \right] + \dots (3)$$

$$\frac{\nu}{c_p} \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B^2(x)}{\rho c_p} u^2 + \frac{Q(x)}{\rho c_p} (T - T_\infty) - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2}\right) - A(C - C_\infty) \quad \dots (4)$$

The boundary conditions are considered as:

$$\begin{aligned} u = U_w, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at } y = \delta(x + b)^{\frac{(1-n)}{2}} \quad \dots (5) \\ u = 0, \quad v = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{at } y \rightarrow \infty \end{aligned}$$

Where,  $u(x, y)$  is the component of velocity in the  $x$ -direction,  $v(x, y)$  is the component of velocity in the  $y$ -direction,  $\nu$  is the kinematic viscosity,  $\beta$  is the Casson parameter,  $\sigma$  is the electrical conductivity of the fluid,  $\rho$  is the density fluid,  $k_0$  is the permeability constant,  $\tau$  is ratio between the effective heat capacity of the nanoparticles material and the heat capacity of the fluid,  $D_B$  is the Brownian diffusion coefficient, is the thermophoresis diffusion coefficient,  $c_p$  is the specific heat coefficient of nanoparticles and  $A$  is the parameter of chemical reaction.

The variable magnetic field  $B(x)$  is taken of the form  $B(x) = B_0(x + b)^{\frac{(n-1)}{2}}$ , while the heat source is represented as  $Q(x) = Q_0(x + b)^{n-1}$ . We assume that the thermal conductivity  $k$  is temperature dependent and can be defined as

$k = k_\infty \left[ 1 + \varepsilon \left( \frac{T - T_\infty}{T_w - T_\infty} \right) \right]$ , where  $\varepsilon$  is the thermal conductivity parameter. Moreover, the radiative heat flux  $q_r$  obeys Rosseland approximation and is defined by:

$$q_r = -\frac{4\sigma^* \partial T^4}{3k^* \partial y} \quad \dots (6)$$

Here  $\sigma^*$  represents Stefan-Boltzmann constant and  $k^*$  is the mean absorption coefficient. We assume that the temperature differences in the flow are small enough so that  $T^4$  can be expressed as a linear function of temperature  $T$  by using Taylor series about  $T_\infty$ . By neglecting the higher order terms, we get:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \quad \dots (7)$$

Now, introducing the following similarities equations so that Eq. (1) is identically satisfied:

$$\psi = \sqrt{\left(\frac{2}{n+1}\right) a \nu (x+b)^{n+1}} F(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \dots (8)$$

$$\eta = \sqrt{\left(\frac{n+1}{2}\right) \frac{a(x+b)^{n-1}}{\nu}} y, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}.$$

where  $\psi$  is the stream function such that  $u = \frac{\partial \psi}{\partial y}$

and  $v = -\frac{\partial \psi}{\partial x}$ .

By applying the similarities (8) to Eqs (2)-(6), the governing equations are reduced into the following dimensionless form:

$$\left(1 + \frac{1}{\beta}\right) F''' + F F'' - \left(\frac{2n}{n+1}\right) F'^2 - \left(\frac{2}{n+1}\right) \left(M + \frac{1}{K}\right) F' = 0 \quad \dots (9)$$

$$\left(\frac{4Rd+3}{3Pr}\right) \theta'' + \frac{\varepsilon}{Pr} (\theta'^2 + \theta \theta'') + F \theta' + N_b \theta' \phi' + N_t \theta'^2 + \dots (10)$$

$$\left(\frac{2\lambda}{n+1}\right) \theta + Ec \left(1 + \frac{1}{\beta}\right) F'^2 + \left(\frac{2M Ec}{n+1}\right) F'^2 = 0$$

$$\phi'' + Le F \phi' + \left(\frac{N_t}{N_b}\right) \theta'' - \left(\frac{2Le\gamma}{n+1}\right) \phi = 0, \quad \dots (11)$$

with the dimensionless boundary conditions:

$$F(\alpha) = \alpha \left(\frac{1-n}{1+n}\right), \quad F'(\alpha) = 1, \quad F'(\infty) = 0, \quad \dots (12)$$

$$\theta(0) = 1, \quad \theta(\infty) = 0, \quad \phi(0) = 1, \quad \phi(\infty) = 0.$$

Here  $M = \frac{\sigma B_0^2}{a \rho}$  is the magnetic field parameter,

$K = \frac{k_0 a (x+b)^{n-1}}{\nu}$  is the permeability parameter,

$Rd = \frac{4\sigma^* T_\infty^3}{k^* k_\infty}$  is the radiation parameter,  $Pr = \frac{\rho c_p \nu}{k_\infty}$  is

the Parndtl number,  $N_b = \frac{\tau D_B (C_w - C_\infty)}{\nu}$  is the Brownian

motion parameter,  $N_t = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}$  is the

thermophoresis parameter,  $\lambda = \frac{Q_0}{a \rho c_p}$  is the heat source

parameter,  $Ec = \frac{a^2 (x+b)^{2n}}{c_p (T_w - T_\infty)}$  is the Eckert number,

$Le = \frac{\nu}{D_B}$  is the Lewis number,  $\gamma = \frac{A}{a(x+b)^{n-1}}$  is the

chemical reaction parameter and  $\alpha = \delta \sqrt{\frac{n+1}{2} \frac{a}{\nu}}$  is the

parameter of thickness.

Note that the primes in the above equations denote the differentiation with respect to  $\eta$ .

By defining  $F(\eta) = f(\eta - \alpha) = f(\zeta)$ , the governing Eqs (9)-(11) and the boundary conditions (12) become:

$$\left(1 + \frac{1}{\beta}\right) f''' + f f'' - \left(\frac{2n}{n+1}\right) f'^2 - \left(\frac{2}{n+1}\right) \left(M + \frac{1}{K}\right) f' = 0 \quad \dots (13)$$

$$\left(\frac{4Rd+3}{3Pr}\right) \theta'' + \frac{\varepsilon}{Pr} (\theta'^2 + \theta \theta'') + f \theta' + N_b \theta' \phi' + N_t \theta'^2 +$$

$$\left(\frac{2\lambda}{n+1}\right) \theta + Ec \left(1 + \frac{1}{\beta}\right) f'^2 + \left(\frac{2M Ec}{n+1}\right) f'^2 = 0 \quad \dots (14)$$

$$\phi'' + Le f \phi' + \left(\frac{N_t}{N_b}\right) \theta'' - \left(\frac{2Le\gamma}{n+1}\right) \phi = 0 \quad \dots (15)$$

with the boundary conditions:

$$f(0) = \alpha \left(\frac{1-n}{1+n}\right), \quad f'(0) = 1, \quad f'(\infty) = 0, \quad \dots (16)$$

$$\theta(0) = 1, \quad \theta(\infty) = 0, \quad \phi(0) = 1, \quad \phi(\infty) = 0.$$

**Numerical solutions**

The numerical technique based on Runge–Kutta–Merson method of order five is employed to solve the system of non-linear differential Eqs (13)-(15) subjected to the boundary conditions (16). To apply shooting method, we use the subroutine D02HAF from the NAG Fortran library which requires the supply of starting values of the missing initial and terminal conditions. In order to control the local truncation error, Runge–Kutta–Merson method with variable step size is used in this subroutine. Then, modified Newton–Raphson technique is applied to make successive corrections to the estimated boundary values. Until convergence is achieved, the process is iteratively repeated multiple times.

The skin-friction-coefficient and reduced Nusselt number in the non-dimensional form can be written as:

$$\tau_\omega = - \left[ \frac{\partial^2 f}{\partial \eta^2} \left( 1 + \frac{1}{\beta} \right) \right]_{\eta=0}, \quad \dots (17)$$

$$Nu = - \left[ \frac{\partial \theta}{\partial \eta} \right]_{\eta=0}. \quad \dots (18)$$

**Results and Discussion**

Numerical computations for the velocity, temperature and nanoparticle concentration profiles have been performed in order to obtain a physical understanding of the problem. The influences of various parameters entering the problem are discussed and illustrated graphically through Fig. 1-7. The values of non-dimensional parameters utilized in the numerical simulations of the graphs are as follows:

$$\alpha = 0.5, \beta = 0.5, n = 0.5, M = 1, K = 1, Rd = 1, Pr = 1, \varepsilon = 0.5, N_b = 0.1, N_t = 0.1, Ec = 0.5, \lambda = 0.5, Le = 0.5, \gamma = 0.1$$

Figs 1 and 2 demonstrate the behaviour of the velocity  $f'(\zeta)$  versus the non-dimensional coordinate  $\zeta$  for varying values of the magnetic field parameter  $M$  and the permeability parameter  $K$ , respectively. It is obvious from these figures that the velocity decreases as  $M$  increases, while it increases as  $K$  increases. The reason behind the result in Fig. 1 is the fact that the influence of the magnetic field on the electrically conductive fluid creates a strong resistance force known as Lorentz force, which slows down the motion of the fluid i.e. decelerates the flow. The result in Fig. 2 is in agreement with the fact that the porous medium reduces the resistance to the flow,

so that as the permeability parameter increases, the medium becomes more permeable, the total mass flow also increases and hence the velocity increases.

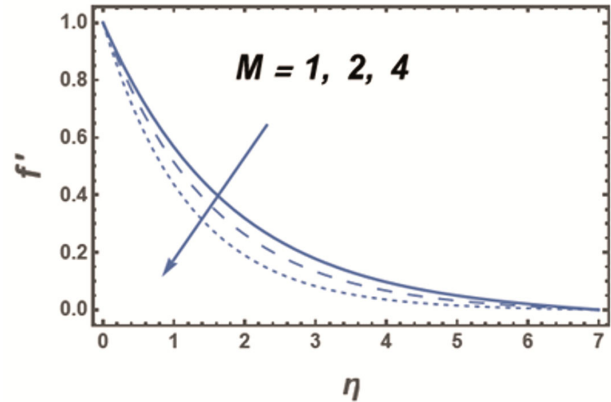


Fig. 1 — Velocity diagram to illustrate the effect of the magnetic field parameter  $M$

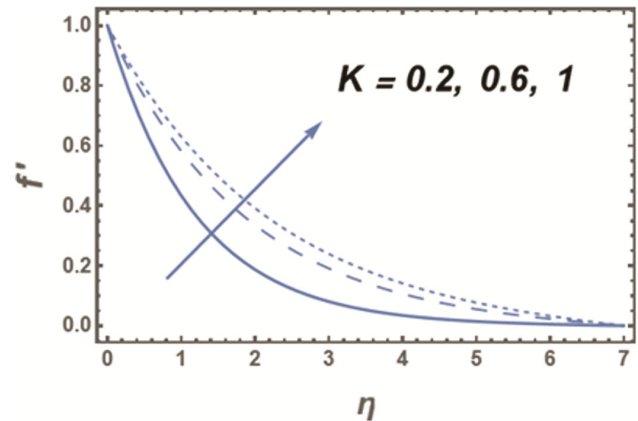


Fig. 2 — Velocity diagram to illustrate the effect of the permeability parameter  $K$

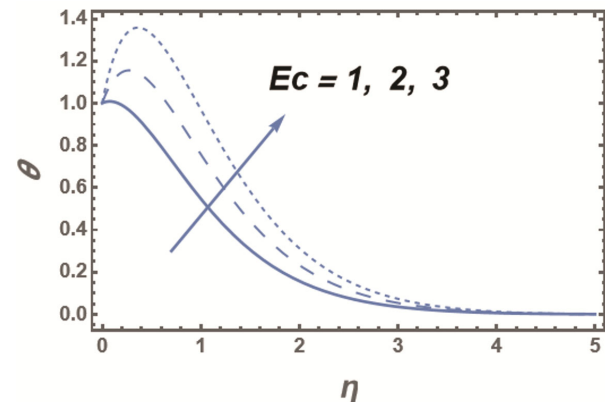


Fig. 3 — Temperature diagram to illustrate the effect of Eckert number  $Ec$

Eckert number denotes the amount of mechanical energy converted to thermal energy via internal friction i.e. heat dissipation. Consequently, increasing values of Eckert number will raise the amount of thermal energy contributing to the flow and the regime will be heated; this will explain the result in Fig. 3. Figs 3 and 4 illustrate the behaviour of the temperature distribution  $\theta(\zeta)$  with the dimensionless coordinate  $\zeta$  for various values of Eckert number  $Ec$  and the thermal radiation parameter  $Rd$ , respectively. It is clear from these figures that  $\theta$  increases as  $Ec$  increases; this is due to the above definitions of Eckert number, while it decreases as  $Rd$  increases. The result in Fig. (4) is because the fact that an increase in the thermal radiation parameter raise the surface heat transfer, rendering the fluid hotter.

Figs 5 and 6 display the variations of the nanoparticles concentration  $\phi(\zeta)$  versus the non-dimensional coordinate  $\zeta$  for several values of the Casson parameter  $\beta$  and the Brownian motion parameter  $N_b$ , respectively. These figures show that  $\phi$  increases by increasing  $\beta$ , while it decreases by increasing  $N_b$ . The following explains the result in Fig. 5; the fluid becomes more viscous with an increase in the Casson parameter, which causes the concentration to increase. The concentration of nanoparticles in the fluid is significantly impacted by Brownian motion. Nanoparticles rearrange due to spontaneous diffusivity, and the thermal conductivity of the nanofluid increases. As a result, the distributions of nanoparticle concentrations are reduced because the larger Brownian motion parameter warms the fluid in the boundary layer and pushes the nanoparticle away from the main flow region into the sheet and this causes reduction in species concentration.

The influence of the chemical reaction parameter  $\gamma$  on the nanoparticles concentration distribution  $\phi(\zeta)$  is illustrated in Fig. 7. It is found that the effect of  $\gamma$  on  $\phi(\zeta)$  is similar to the impact of  $N_b$  on  $\phi(\zeta)$  given in Fig. 6, with the only difference that the curves in Fig. 7 are very close to each other than those obtained in Fig. 6. Now, we will explain how the chemical reaction parameter affects the nanoparticles concentration. Increasing the chemical reaction parameter (gamma) causes an increment in particles' interfaces, and then nanoparticles

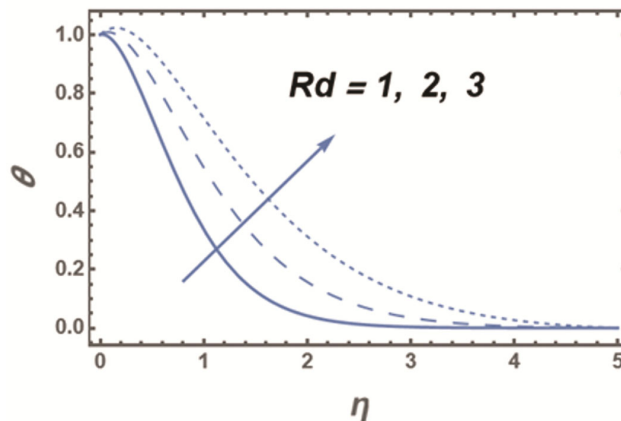


Fig. 4 — Temperature diagram to illustrate the effect of the thermal radiation parameter  $Rd$

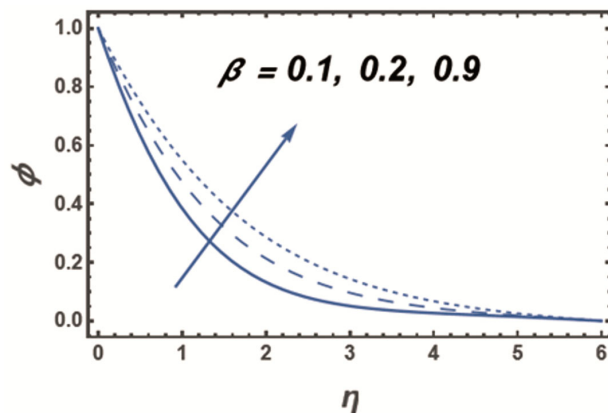


Fig. 5 — Nanoparticles concentration diagram to illustrate the effect of the Casson parameter  $\beta$

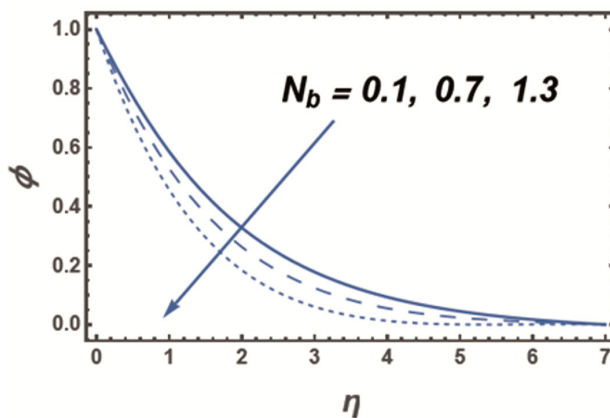


Fig. 6 — Nanoparticles concentration diagram to illustrate the effect of the Brownian motion parameter  $N_b$

concentration will decrease. The impacts of other parameters are similar to those obtained in Figs. (5) and (6). But, they are excluded here to avoid any kind

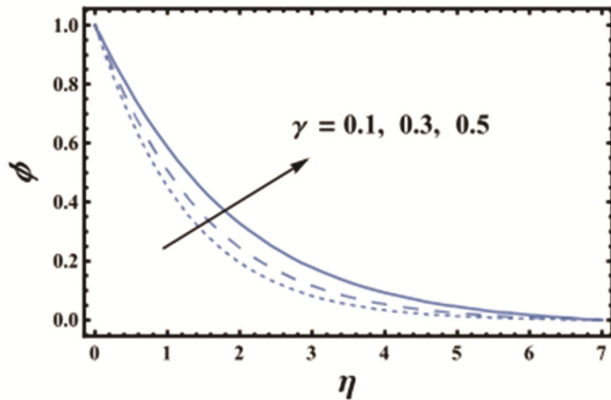


Fig. 7 — Nanoparticles concentration diagram to illustrate the effect of the chemical reaction parameter  $\gamma$

Table 1 — Both skin friction and Nusselt number for different values of  $\alpha$  and  $\gamma$

$\alpha$	$\gamma$	The skin friction	Nusselt number
0.2	0.1	-0.7370	-0.4505
0.6	0.1	-0.2816	-0.3816
1	0.1	-0.0642	-0.2042
1	0.3	0.0151	-0.9152
1	0.5	0.0924	-1.0381

of repetition.

Table 1 presents numerical results for the skin friction  $\tau_w$ , and reduced Nusselt number  $Nu$ , for various values of the thickness parameter  $\alpha$ , and the chemical reaction parameter  $\gamma$ . It is clear from Table 1, that an increase in  $\alpha$  gives an increase in both values of the dimensionless quantities  $\tau_w$  and  $Nu$ . But when  $\gamma$  increases,  $Nu$  decreases, while  $\tau_w$  increases.

### Conclusion

In this paper, the MHD flow of Casson nanofluid with variable thermal conductivity past a moving surface of variable thickness is investigated. The effects of thermal radiation, heat source, Ohmic heating, viscous dissipation and chemical reaction are included in this investigation. The governing PDEs are converted to third-order ODEs by the similarity transformation. The velocity decreases by increasing the magnetic field parameter  $M$ , while it increases at high values of  $K$ . An increase of Eckert number  $E_c$  and a decrease of the thermal radiation parameter  $R$  cause an increase of the temperature. The nanoparticles concentration growth at high values of the Casson parameter  $\beta$ , while it decreases as the

Brownian motion parameter  $N_b$  increases. Some interesting results about MHD Mixed convection Couette flow of biviscosity nanofluid that are relevant to micro-tribological designs have been obtained by this study; however, it can be extended in the future by taking into account the cases of non-Newtonian models for ionic magnetic lubricants, like microstructural, viscoelastic, etc., under the influence of quadratic thermal radiation flux, generalized mass, and thermal convection with Cattaneo-Christo heat flux when developing nano-electro magneto hydrodynamic transporting devices for pharmacology, it is important to minimize the amount of entropy produced.

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