Supplementary Information

Modeling of magnetohydrodynamic Casson nanofluid flow with nonlinear thermal radiation in a stretching porous channel using ANN and Levenberg–Marquardt optimization

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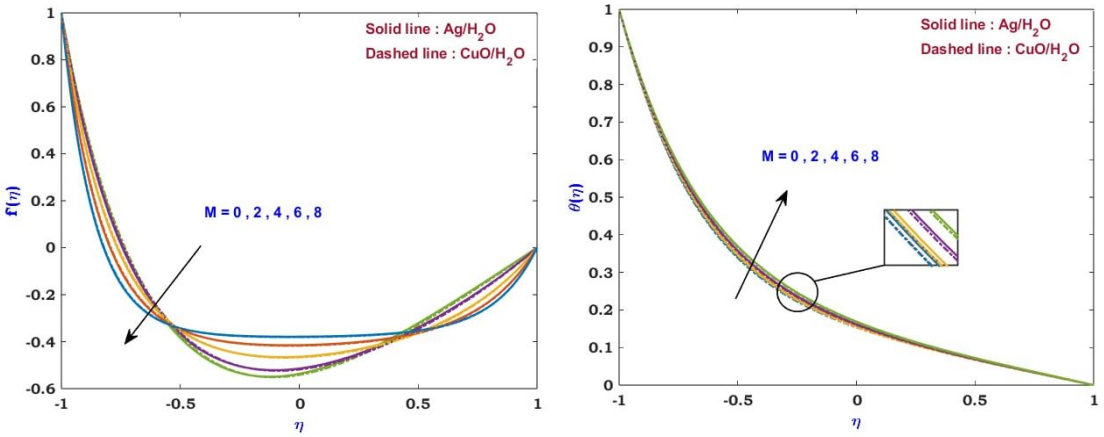

Fig. S1 — Impacts of M versus velocity (a) f′ and (b) θ

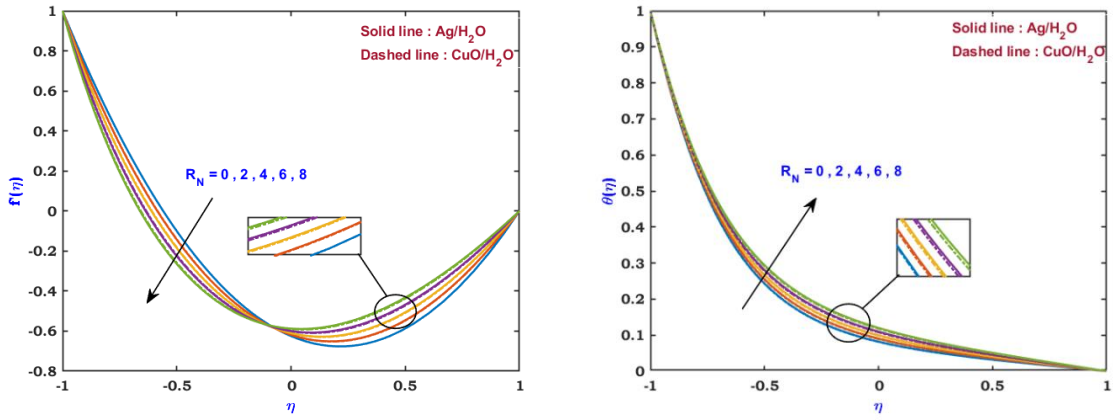

Fig. S2 — Impacts of R_N on (a) f′ and (b) θ

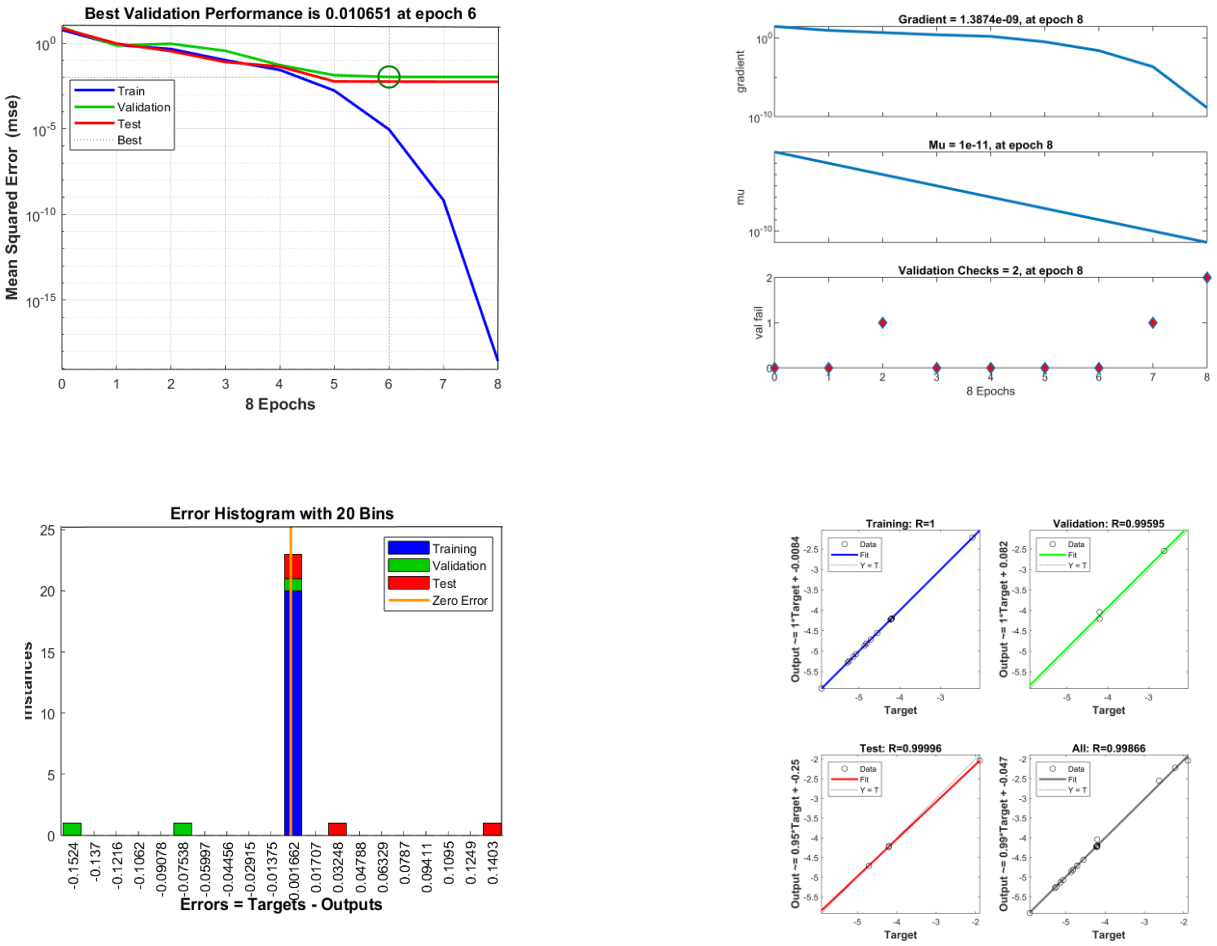

Fig. S3 — ANN on Skin Friction values of *Ag-H_2O*: (a) performance analysis, (b) Training se, (c) error histogram and (d) regression analysis

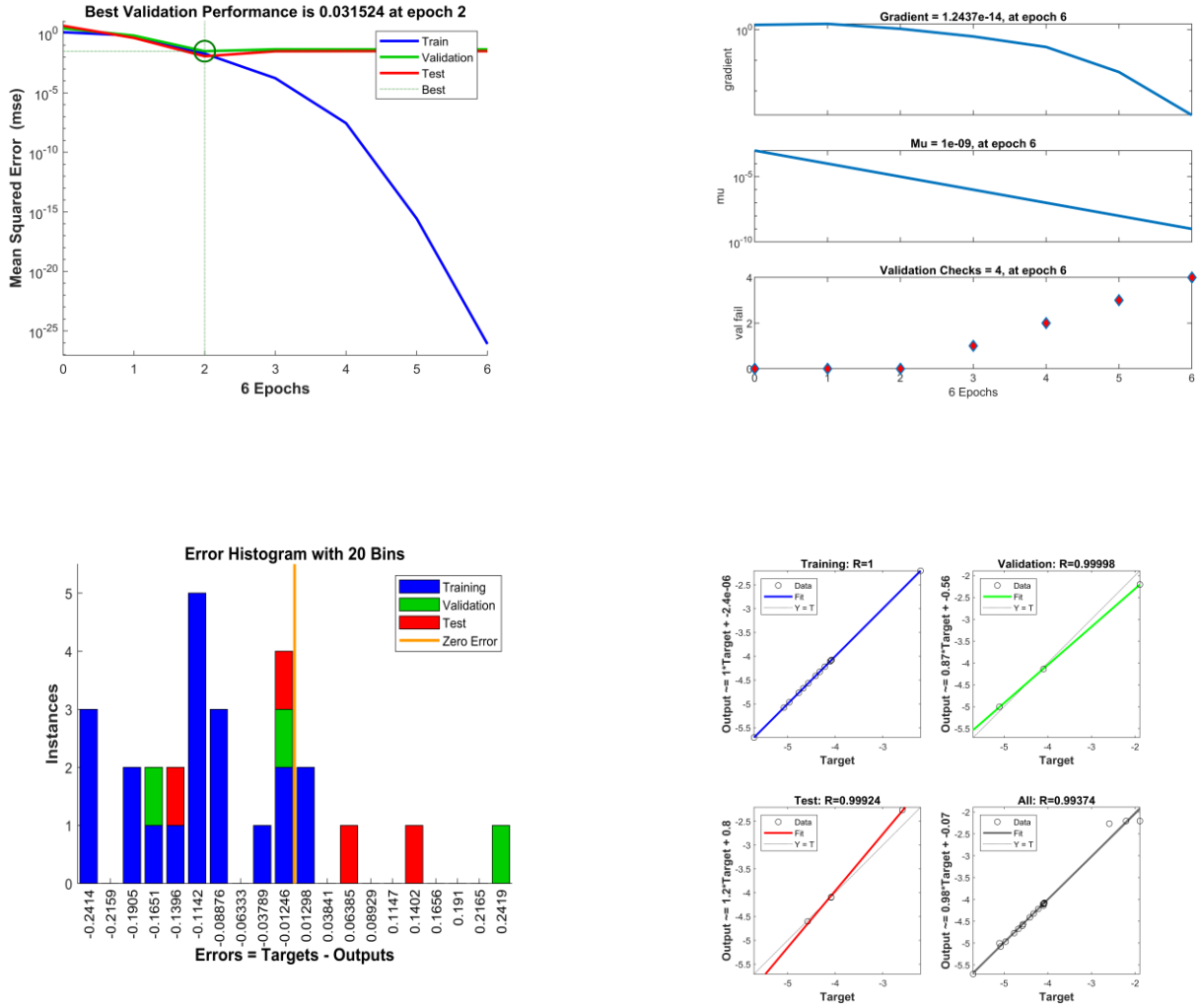

Fig. S4 — ANN on Skin Friction values of CuO-H_2O: (a) performance analysis, (b) Training set, (c) error histogram and (d) regression analysis

Table S1 — Skin friction values of Ag/H_2O for different values of parameters											
β_c	ϕ	***Pr***	R_N	λ	**M**	**N**	H_T	K_r	**τ values** ***Ag/H₂O***	**ANNvalues** ***Ag H₂O /***	**Error values** ***Ag/ H₂O***
1.0	**0.01**	**1**	**10**	**0.5**	**0.4**	**0.3**	**-0.5**	**0.5**	-4.202	-4.2080	**0.0060**
1.5									-4.818	-4.8192	**0.0012**
2.0									-5.243	-5.078	**0.0010**
	0.03								-4.552	-4.5564	**0.0044**
	0.05								-4.864	-4.8664	**0.0024**
	0.07								-5.136	-5.1358	**-0.0001**
		3							-4.202	-4.2057	**0.0037**
		5							-4.202	-4.2030	**0.0010**
		7							-4.202	-4.0418	**-0.1601**
			4						-2.625	-2.5456	**-0.0793**
			2						-2.220	-2.2209	**0.0009**
			0						-1.891	-2.0390	**0.1480**
				0.6					-4.709	-4.7122	**0.0032**
				0.7					-5.279	-5.2778	**-0.0011**
				0.8					-5.917	-5.9127	**-0.0042**
					2.0				-4.708	-4.7078	**-0.0001**
					4.0				-5.071	-5.0732	**0.0022**
					6.0				-4.819	-4.8162	**-0.0027**
						0.5			-4.202	-4.2064	**0.0044**
						0.7			-4.202	-4.2063	**0.0043**
						0.9			-4.202	-4.2031	**0.0011**
							-0.4		-4.202	-4.2067	**0.0047**
							-0.3		-4.202	-4.2321	**0.0301**
							-0.2		-4.202	-4.2045	**0.0025**
								0	-4.226	-4.2275	**0.0015**
								0.3	-4.212	-4.2160	**0.0040**
								0.7	-4.193	-4.1981	**0.0051**

Table S2 — Skin friction values of CuO/H_2O for different values of parameters											
β_c	ϕ	*Pr*	R_N	λ	M	N	H_T	K_r	τ values *CuO/* ***H₂O***	ANN values *CuO/* ***H₂O***	Error values *CuO/* ***H₂O***
1.0	0.01	1	10	0.5	0.4	0.3	-0.5	0.5	-4.087	-3.9747	-0.1122
1.5									-4.673	-4.4385	-0.2344
2.0									-5.078	-4.9720	-0.1059
	0.03								-4.218	-4.0925	-0.1254
	0.05								-4.326	-4.4628	0.1368
	0.07								-4.413	-4.6676	0.2546
		3							-4.087	-3.9151	-0.1718
		5							-4.087	-3.8854	-0.2015
		7							-4.087	-3.8329	-0.2540
			4						-2.59	-2.4552	-0.1347
			2						-2.205	-2.1231	-0.0818
			0						-1.89	-1.8930	0.0030

				0.6					-4.567	-4.4838	-0.0831
				0.7					-5.106	-5.0851	-0.0208
				0.8					-5.708	-5.6573	-0.0506
					2.0				-4.58	-4.3919	-0.1880
					4.0				-4.966	-4.7170	-0.2489
					6.0				-4.769	-4.5987	-0.1702
						0.5			-4.087	-4.0660	-0.0209
						0.7			-4.087	-4.0728	-0.0141
						0.9			-4.087	-4.0801	-0.0068
							-0.4		-4.087	-4.1618	0.0748
							-0.3		-4.087	-4.0952	0.0082
							-0.2		-4.087	-3.9377	-0.1492
								0	-4.11	-3.9949	-0.1150
								0.3	-4.096	-4.0119	-0.0840
								0.7	-4.078	-3.9672	-0.1107

Table S3 — Nusselt number of Ag/H_2O for different values of parameters

β_c	**ϕ**	***Pr***	R_N	**λ**	**M**	**N**	H_T	K_r	***Nu* values Ag/ H_2O**	**ANN values Ag/ H_2O**	**Error values Ag/ H_2O**
1.0	**0.01**	**1**	**10**	**0.5**	**0.4**	**0.3**	**-0.5**	**0.5**	-0.975	-0.9777	**0.0027**
1.5									-0.967	-0.9710	**0.0040**
2.0									-0.963	-0.9681	**0.0051**
	0.03								-0.945	-0.9474	**0.0024**
	0.05								-0.918	-0.9200	**0.0020**
	0.07								-0.894	-0.8952	**0.0012**
		3							-1.953	-1.5612	**-0.3917**
		5							-2.91	-4.5112	**1.6012**
		7							-8.936	-8.9363	**0.0003**
			4						-0.999	-1.0074	**0.0084**
			2						-1.007	-1.0172	**0.0102**
			0						-1.015	-1.0043	**-0.0106**
				0.6					-1.01	-1.0201	**0.0101**
				0.7					-1.046	-1.0491	**0.0031**
				0.8					-1.082	-1.0871	**0.0051**
					2.0				-0.962	-0.9284	**-0.0335**
					4.0				-0.949	-0.9522	**0.0032**
					6.0				-0.943	-0.9450	**0.0020**
						0.5			-0.912	-0.9136	**0.0016**
						0.7			-0.863	-0.8639	**0.0006**
						0.9			-0.825	-0.9356	**0.1106**
							-0.4		-0.932	-0.9338	**0.0018**
							-0.3		-0.888	-0.8891	**0.0011**
							-0.2		-0.843	-0.8439	**0.0009**
								0	-0.975	-1.0353	**0.0603**
								0.3	-0.975	-0.9758	**0.0008**
								0.7	-0.975	-1.0196	**0.0446**

Table S4 — Nusselt number of CuO/H_2O for different values of parameters											
β_c	**ϕ**	***Pr***	R_N	**λ**	**M**	**N**	H_T	K_r	***Nu* values *CuO/H_2O***	**ANN values *CuO/H_2O***	**Error values *CuO/H_2O***
1.0	**0.01**	**1**	**10**	**0.5**	**0.4**	**0.3**	**-0.5**	**0.5**	-0.978	-0.9790	**0.0010**
1.5									-0.97	-0.9421	**-0.0278**
2.0									-0.966	-0.9751	**0.0091**
	0.03								-0.953	-0.9512	**-0.0017**
	0.05								-0.93	-0.9298	**-0.0001**
	0.07								-0.908	-0.9077	**-0.0002**
		3							-1.961	-1.9616	**0.0006**
		5							-2.922	-2.9659	**-0.0008**
		7							-3.845	-3.8452	**0.0002**
			4						-1.001	-1.0024	**0.0014**
			2						-1.009	-1.0102	**0.0012**
			0						-1.016	-1.0170	**0.0010**
				0.6					-1.013	-1.0154	**0.0024**
				0.7					-1.049	-1.0505	**0.0015**
				0.8					-1.086	-1.0836	**-0.0023**
					2.0				-0.965	-0.9474	**-0.0175**
					4.0				-0.951	-0.9573	**0.0063**
					6.0				-0.945	-1.0568	**0.1118**
						0.5			-0.914	-0.9137	**-0.0002**
						0.7			-0.865	-0.8785	**0.0135**
						0.9			-0.827	-0.8254	**-0.0015**
							-0.4		-0.935	-0.9354	**0.0004**
							-0.3		-0.891	-0.8774	**-0.0135**
							-0.2		-0.845	-0.8469	**0.0019**
								0	-0.977	-0.9846	**0.0076**
								0.3	-0.978	-0.9818	**0.0038**
								0.7	-0.978	-0.9719	**-0.0060**

Table S8 Parameter values of ANN models	
Parameters	Values
Training algorithm	LM
Number of neurons in input layer	9
Number of neurons in hidden layer	10
Number of neurons in output layer	2
Learning rate	0.1
Momentum term	0.1
Error tolerance	0.001
Maximum epochs	50
Number of epochs	6
Training speed (Sec)	00.01